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# mathetics

DEVOTED TO APPLICATIONS OF BEHAVIOR ANALYSIS

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CONTENTS



*Scheduled for publication in this issue, an original paper by* Israel Goldiamond: Maintenance of Ongoing Fluent Verbal Behavior and Stut-tering, *has been postponed for reasons beyond control of Dr. Goldiamond and of the editors. This paper will appear in the next issue.*

# EDITORIAL

The conviction has prevailed that the principles of human learning are simply a collection of conflicting points of view. But the principles of learning are no longer matters of viewpoint. An authoritative account of the major events relevant to learning is contained in the program for investigating animal behavior developed by B. F. Skinner of Harvard and commonly called reinforcement theory or operant conditioning. This system is receiving increasing attention despite the popularity of Freudian psychology, the mental test movement, and the proclivity of psychologists for electicism.

The teaching machine, which has popularized reinforcement theory, draws upon the notion of learning by doing. However, the theoretical system as a whole remains to be exploited for human betterment. Programmed education bears more relation to Thorndike than to the modern science of animal behavior. In the field of self-instruction, Skinner's central visions have been overlooked.

The success of reinforcement theory in developing complex behavior repertories in animals has led us to see the relevance of this science to the problems of training the human mind. The human is subject to the same laws that govern the behavior of other animals. Indeed, it appears that the higher the organism the more efficient the application of reinforcement and the wider the range of reinforcers. While success with animals does not automatically mean success with human beings, there is a well developed body of fact and experience in applying reinforcement theory to human training.

The purpose of *The Journal of Mathetics* is to communicate both the accomplishments and the potentialities of a technology of education made possible by this authoritative science of learning.

Because the success of the methods of operant conditioning is due in part to orientations not typical of the classical approaches of psychology, a description of three essentials which contrast with current attitudes toward behavior study may well be an appropriate introduction to this *Journal*:

*Study of individuals rather than statistics*

The methods of operant conditioning resemble those of clinical study. The same organism is studied day after day, perhaps for hours at a time. The accumulation of base line data makes the subject his own control. His previous and unique history will determine most of his behavior in a manner unknown to the experimenter. Detailed records are made of a specific behavior under study. Cumulative recordings are possible, revealing a continuous picture of frequencies with which a given sample of behavior is emitted. Whereas summations and totals would tend to conceal significant findings, individual records afford fine-grain analysis of moment-to-moment variations, and stable differences between subjects are observed in detail rather than averaged.

*Complete freedom of the subject's behavior*

With minimal instruction—or even with none—the subject himself discovers the consequences of his own behavior. This freedom has led Skinner to refer to such behavior as the free operant. Conditions controlling the subject arise from his own initiative. The operant conditioner always begins with behavior the subject himself initiates. Without prior concept of how he ought to behave, the examiner observes how he does indeed behave when free to behave naturally. It is interesting that the methods of operant conditioning are common to the clinical methods of psychoanalysis and nondirective therapy with respect to intensive study of the individual and the free expression of the subject.

*Emphasis on control rather than prediction*

Much scientific study pertinent to learning has been directed to the prediction of how a specific organism, whether it be a pigeon or a college sophomore, will learn to perform a standardized task. Such emphasis may well lead to the prediction that variable A is more efficient than variable B while obscuring the inefficiency of both variables. In operant conditioning laboratories the emphasis has been away from prediction in this sense. Emphasis is rather on discovering what conditions will produce some

> particular behavior. The educator—whether school teacher, therapist, or trainer—is faced with the task of modifying behavior, not with prediction. The laboratory concentrating on this task is prepared to find bases for new tools by which the task may be accomplished—tools which may override the effects of agencies operating on the subject prior to his coming to the educator.

Too frequently in the history of research relevant to education and mental hygiene, the orientation has been toward some specific technique. *The Journal of Mathetics* is not so preoccupied. While research must always be in the business of perfecting techniques, and the *Journal* in the business of reporting them, the precedence of techniques over problems to be solved tends to emphasize a secondary aim of science. The primary questions lie in the realm of human need, and first among these is the problem of ordering human behavior.

This long road of educational research divides into many branches. In recent decades some of these paths have led to new fields of discovery. It is the purpose of this *Journal* to give definition to one such field which is both new and rich in promise. The editors hope to assist their colleagues—the men and women of science and education working in this field—in keeping abreast of its development and contributing to its knowledge.

# MATHETICS: THE TECHNOLOGY OF EDUCATION[1]

BY THOMAS F. GILBERT[2]
*TOR Laboratories, Inc.*[3]

EDUCATION is one major commerce in which men depend more upon contrivance than upon design; there has existed no genuine technology of training humans. Technology is a systematic application of science to the industries of men, and not until recent years has the science directly relevant to education possessed the dependable and simple order necessary for systematic application. The system of behavior analysis formed largely by B. F. Skinner represents a scientific theory of sufficient maturity to serve as a foundation for a true technology of education. Reinforcement theory has made possible efficient and methodical control of animal behavior. Man is more than animal, and a technology of control will not supply wisdom about *what* to teach and *whom* to teach; nevertheless these principles of control do apply, for man is at least an animal whatever else he may be. If the control of the human animal is the chief commodity of education and if the basic principles of control are adequate, an applied science of teaching awaits only the parts being put together.

My purpose here is to make public for the first time the general structure of a scientifically based educational technology that my colleagues and I call *mathetics*,[4] a name given to emphasize its uniquely systematic quality and to distin-

1. This is the first of a series of papers that will appear in successive issues of the *Journal*. The remainder of the series will be devoted to details of the general system described here.
2. The true authors of this series are my colleagues at TOR Laboratories, whose work my words are all about.
3. This work was supported in part by the University of Alabama School of Dentistry with funds made available by Dr. Joseph Volker, Dean.
4. From the Greek *mathein*, meaning "to learn". Mathetics is not a coined word and appears in the unabridged *Webster's New International Dictionary*, 2nd Ed.

guish it from less systematic methods of planning the course of human learning. Mathetics may be defined as the systematic application of reinforcement theory to the analysis and reconstruction of those complex behavior repertories usually known as "subject-matter mastery," "knowledge," and "skill." Mathetics, like any technology, requires both an understanding of its principles and apprenticeship in conditions of the field to which it applies. Although this series of treatises is theoretical the system it describes developed gradually out of actual experience, and I no longer hold any doubt that it is a true and practicable technology, so accurate in the main that time can exact changes only toward greater parsimony and the enrichment and correction of subordinate detail.

A genuine applied science has certain features which, in the sum, distinguish it from intuitive artistries and from the fundamental sciences on which it is based. Technological principles are derivative, not fundamentally new. They are adaptations of existing scientific principles to the conditions uniquely characterizing the commerce to which that technology is applied. Intuitively "clever" ideas are not put into effect until their relation to the theory is understood, or when they merely serve to augment the basic principles. Mathetics has these features. Its principles are few in number and are taken literally from the science of animal behavior. Its additional procedural rules are based on observation, but we have learned to suspect our observations when they did not honor the theory, for more meticulous application has proved the theory right.

Mathetics, if applied diligently, produces teaching materials that exceed the efficiency of lessons produced by any known method. We have compared mathetical exercises with the best available programmed learning materials and found that these programs require twice to ten times as much learning time, five to thirty times as many exercises to cover the same subject-matter, and the programs result in poorer recall. The more difficult the material, the greater the advantage gained by mathetics. Mathetics is applicable to all subject-matters, and has been used to train delicate manual skills as well as teach complex verbal repertories. Less cost and less time are required to produce mastery by mathetics; most matheticists are young women without graduate degree, and some without undergraduate degree. Matheticists do not have

to be experts in the subject-matter for which they design lessons, and expert consultation seldom requires more than one percent of the total time of production. Two matheticists working independently on the same subject matter will produce lessons that are virtually identical in all essential respects. Mathetics is not limited to educational subject-matter; it has been applied to varieties of problems ranging from the control of smoking behavior to the management of the young child in the home. The reader who moves carefully through this treatise will be persuaded that these claims are not excessive. Mathetics is not just another point-of-view about teaching and learning; it is an authoritative scientific technology.

*General Plan of the Treatise*

This treatise is intended only as a description of the formal system of mathetics. The many practical details relevant to the production of teaching lessons will be described in later articles of this series. The examples used in this paper are taken from actual work, but they have been modified for purposes of illustration and should not be considered complete or accurate in detail.

Mathetics is a production process consisting of several distinctive stages. In summary, these are:

(a) *Prescription*, a description of the behaviors that constitute mastery in some subject-matter domain. The only behaviors made explicit in the prescription are those necessary to synthesize mastery performance. The behavior repertory that the prescription represents is called the *synthetic repertory* and is roughly equivalent to what is meant by the word "practice" as distinguished from "theory."

(b) *Development of the Domain Theory.* In this process the essential elements of the subject-matter are extracted from the details and described in behavior terms. A domain theory is relevant only to the subject-matter reflected in the prescription. A new prescription is written describing the analytic observations of a subject-matter master; this

is the behavior we usually mean when we say a student has "understanding" or knows "theory." This new prescription describes the *analytic repertory.*

(c) *Characterization.* In this process an analysis is made of the behavior properties of the prescribed repertories; a description is made of the generalizations that are to be taught, of competition with adequate performance, and of the behavior available to overcome this competition. It is from this information that a plan of the lesson is developed.

(d) *Exercise Design.* The exercises are designed according to a basic model, and in a sequence consistent with a lesson plan developed from the characterization.

In the sections that follow, the exercise model is first described, and the other mathetical stages are treated in the order given above. A glossary of special terminology is provided. Some terms are used with such frequency, however, that the reader may need the definitions given below.

(a) *Chain.* Behavior can be thought of as a sequence of events having three properties: a *response*, some occasion for that response (called a *stimulus*), and some consequences that makes the response more or less likely to occur again on the same occasion. If the effect produced by the response makes it *more* likely to occur again, that effect is called a *reinforcer.* A behavior chain is a sequence of stimuli, responses, and reinforcers in which the reinforcer for one response is also the stimulus for the next response. We symbolically represent a chain in this way:

$$S \longrightarrow R \;\cdot\; S \longrightarrow R \;\cdot\; S \longrightarrow R$$

| S | R | S | R | S | R |
|---|---|---|---|---|---|
| Vending machine | put a nickle in | candy comes out | put candy in mouth | sweet taste | chew |

In this example the first stimulus (S) sets the occasion for the first response; the response is reinforced by the appearance of candy which be-

comes the stimulus for the second response. The reinforcer of the second response is the stimulus for the third response. The unity of the behavior chain is determined by this reciprocal relation in which responses produce stimuli and stimuli produce responses. Neither response nor stimulus alone has any behavioral meaning unless viewed in relation to each other.

(b) *Operants and acts.* Behavior must be viewed as a relation between stimulus and response; a response considered by itself has no behavioral meaning—the movement of the wrist has one meaning if it results in a slap, another meaning if it makes a ball bounce. When we analyze behavior we have to find some unit, a building block from which behavior is constructed. That behavior unit must be a relation between stimulus and response. There are two relations that we can look at: we can look to see what stimulus is producing a response, or we can look to see what stimulus is produced by a response. We symbolize the first of these units in this way: $S \longrightarrow R$. and this unit is called an *operant.* We apply brakes when we see a red light, and the response is specifically under the control of the red light; applying brakes at a red light is an operant. When we apply brakes the car stops, and this stopping is under the specific control of the response. When we look at the behavior to see what the response produces, we are looking at it from a different point of view than the operant represents, and this unit I call an act. An act is symbolized as $R \cdot S$.

(c) *Repertory.* The collective behaviors existing at strength in an animal at any given time. The *initial repertory* refers to the available behavior prior to instruction, the *mastery* (or terminal) repertory refers to the behaviors constituting skill in the subject-matter.

(d) *Use of superscripts.* Since stimuli are employed for many purposes, it is convenient to distinguish these purposes by using an appropriate superscript with the symbol for stimulus (S). For example, $S^0$

means a stimulus used to get a student to look at something, to make an *observing* response. The superscript, $^{O}$, simply stands for 'observing." $S^{D}$ is a stimulus that the student must learn to *discriminate* in mastery performance; it is not a stimulus used for teaching purposes, but is one inherent in mastery. Other superscripts are explained in the text.

## PART I. INTRODUCTION

Educational technology deals with two logically independent properties of the learning process. In the education profession these have been most often identified as "knowledge" and "motivation"; in the language of reinforcement theory we can translate these as "stimulus control" and "reinforcement." Briefly the problem of teaching is (a) to determine what steps a student must take to master a subject, and (b) to arrange the conditions that will insure his taking these steps.

Motivation and stimulus control are independent properties of learning. This is established by the fact that it is not sufficient for an animal to be motivated for him to learn. A novice with a hungry pigeon in a box and an efficient food magazine will probably fail to teach the pigeon a simple behavior chain. He does not know how to use the reinforcer in proper conjunction with the available stimuli. A trained operant-conditioner, however, can use any convenient reinforcer to create complex repertories in the pigeon. It is because of this independence of motivation and stimulus control that it is convenient to consider them separately.

### *Motivation*

Certain assumptions about motivation need examination, for they underlie the thesis developed here. The principles and procedures I describe will assume a motivated student, one that possesses a genuine educational objective. If the animal is not motivated, or to put it differently, if the teaching agent is not a clear instrument by which he can achieve a reinforcer, the animal will not perform and cannot learn. If a student is

to learn, the consequences of the mastery of a knowledge or skill must be reinforcing.

We must assume that the student possesses an educational objective, because by itself frequent and apparent success in the course of learning is not *intrinsically* reinforcing. Inherent in any well-designed set of teaching materials is an attempt to maintain the motivation of the student. No matter how well designed materials are, if mastery in a subject is not an objective of the student, or those who control him, the materials will fail because the student will not complete them. People are circumspect in what they choose to learn; the so-called programming principle that progress through the course of learning is inherently reinforcing is not only poor learning theory, but is, I think, an abandonment of common sense. We confuse the tendency to pay homage to education with a desire to learn. Only seven percent of those who buy a well-known encyclopedia ever open its pages; this multivolume symbol of culture is placed on the altar of the home next to the even more infrequently used *Holy Bible*. We learn for the sake of attaining more fundamental objectives; sensible people do not hunger after useless assortments of information. The objective of learning arithmetic is usually established in the child by the society of the child's peers, principally in the school room, and getting a child to learn arithmetic at home in the absence of that society would be even more difficult than getting him to keep his toys in order. Entertainment, by definition, is fun for entertainment's sake; it is valued because it protects the normal behavior patterns that physical or mental fatigue has weakened. Learning, on the other hand, is a *change* in behavior, and the animal—man not excepted—resists that change until some purpose is promised beyond the mere change itself.

If the student's objectives are important in the design of a successful teaching sequence, more immediately important are the objectives of those who pay the educational bills. Education is that strange commerce in which the consumer is seldom the customer. The student submits to a behavior change for those of his purposes it will serve—increased income, status, avoidance of disapproval, removal of punishment, or the attainment of means to new experiences. Those who pay for his behavior change, whether parents or employers, desire the change for reasons of their own, and for

reasons that may not coincide with those of the student. The student is in a real sense an employee of those who support education. The behavior he acquires is valuable to others, whether for noble or vulgar reasons, and their support is likely the most important determinant of the student's own objectives.

Motivation is not a variable in mathetics; it is a fixed constant upon which all the other principles are based. Two assumptions about motivation are fundamental to mathetics. One is that the student is sufficiently motivated by the benefits of mastery to give the minimum effort necessary to achieve that mastery. The other assumption is that the behavior change produced in a learning sequence is sufficiently valuable to people *other* than the student that they are willing to pay for it. This second assumption will keep before us the distinction between materials designed for entertainment and materials designed for education. If no one but the student will pay for the materials, it is likely that others place small value on the behavior change these materials produce; therefore we do not have a reliable source of motivation and must depend upon the entertainment of the student.

Three other precepts that underlie the system are summarized here. These are developd in more detail at appropriate places in the treatise:

(a) All the acts of mastery exist in the initial repertory of any student. Learning does not put something into a student; it is the process of *rearranging* his repertory so that acts will occur in a different sequence and on different occasions than they do in his initial repertory. This precept is basic to reinforcement theory; if an operant is strengthened by its consequences, the response must be in the repertory to produce the consequence. This precept is equivalent to John Dewey's definition of education as the "reconstruction of experience," another way of saying "rearrangement of the repertory."

(b) It is a corollary of the first precept that individual differences among students prepared to enter a given course of instruction are small compared with the communalities. Differences between the

operants in two repertories can be small, while the differences between the *effects* of the two repertories can be large. Imagine a man who is a perfect marksman in every respect but one: he does not know where the rifle's trigger is. The difference between the number of operants in his repertory and the number in the repertory of the finished master is small but the effect of the difference is great.

(c) The principles that apply to overt behavior apply equally to covert behavior; actions of an animal that cannot be observed directly are still behaviors. Covert behavior can be examined and controlled by arranging for it or its effects to become overt. Thinking is behavior, too, and subject to the principles of reinforcement.

## PART II. THE EXERCISE MODEL

The several phases of mathetics result in a *lesson plan*, and this lesson plan is the basis for the last formal mathetical step: writing the exercises. The lesson plan is a precise guide to the exercise writer of such sufficiency that practically nothing is left to guesswork or imagination. The lesson plan is a formal specification of exercise structure and content.

Every exercise has the same basic structure, regardless of content. This structure has its basis in behavior principles. To understand the exercise model one must have an elementary grasp of the behavior principles of *chaining* and *conditioned reinforcement*. Although these principles are quite simple and obvious when understood, certain of their implications are so contrary to common thought that one must approach their understanding with care.

The nature of behavior chaining can be illustrated by a simple animal training problem. Suppose we wish to train a rat to perform, in sequence, these behaviors:

1st operant: Pull a lamp cord to light a light ($S_1 \longrightarrow R$)

2nd operant: Press a lever until a buzzer sounds ($S_2 \longrightarrow R$)

3rd operant: Go to a food pan ($S_3 \longrightarrow R$)

This behavior chain can be represented in stimulus-response notation as:

$$S_1 \longrightarrow R \;\cdot\; S_2 \longrightarrow R \;\cdot\; S_3 \longrightarrow R \;\cdot\; S_4 \ldots$$

| $S_1$ | $R$ | $S_2$ | $R$ | $S_3$ | $R$ | $S_4$ |
|---|---|---|---|---|---|---|
| Lamp cord | Pull cord | Light on | Press lever | Buzzer sounds | Approach pan | Food pellet |

Asked how to train the rat to perform this sequence, the novice will advise us to begin by teaching the animal to pull the lamp cord; it is a well established habit to begin things at the beginning. In practice, however, the novice probably would never succeed in establishing this sequence. What would reinforce the animal even if he pulled the cord? A light has little value to the rat. If cord-pulling were reinforced by feeding the animal, we would violate the sequential restriction which requires the animal to press a lever before approaching the pan. It is well established in practice and in theory that the obvious approach of beginning at the beginning of the sequence is extremely inefficient; months and even years may be added to the time it should take to train an animal to perform such a simple sequence.

We properly begin training our animal by starting with some act of mastery that is reinforcing to the animal. In the example we would begin with the act of approaching the food pan and finding food ($R \cdot S_4$). We then teach the animal that food will not be found unless a buzzer is sounding. He learns that whenever he approaches the pan with the buzzer sounding, he gets reinforced. Now we have established this much mastery: $S_3 \longrightarrow R \cdot S_4$. Next, we provide for the lever to sound the buzzer; whenever the rat, in exploring the cage, presses the lever, he gets *immediately* reinforced. The immediate reinforcer is the buzzer itself; it is now reinforcing because it has become the occasion for approaching the food pan. Much of behavior science is based on this simple fact: an event that normally has no value to an animal will become reinforcing when it sets the occasion for a reinforcing act; we say that this event has become a *conditioned reinforcer*. With the conditioned reinforcement of the response of pressing the lever, we have established: $R \cdot S_3 \longrightarrow R \cdot S_4$. Next we teach the animal that the buzzer will not sound when the lever is pressed unless a light is on. Now we have $S_2 \longrightarrow R \cdot S_3 \longrightarrow R \cdot S_4$. It should be obvious how the remainder of the chain is

produced, and that the light has become reinforcing by virtue of being an occasion for the buzzer working.

The principle of chaining derives from a fundamental of animal nature: You cannot teach an animal to do anything not already in his repertory. The word education is etymologically quite correct; existing behavior is *drawn out* of the animal and made more likely to occur again by consequences that are reinforcing. We establish the occasion ($S^D$) on which this behavior gets reinforced, then make it possible for the animal to produce this occasion, and so on through the chain.

*Writing An Exercise*

Consider a simple behavior chain such as performing long division. This behavior is represented in stimulus-response notation in Figure 1. The behavior has been oversimplified for illustrative purposes and letters are used to represent actual numbers. We shall assume the student knows short division, basic terminology, what to do with remainders and how to estimate quotients.

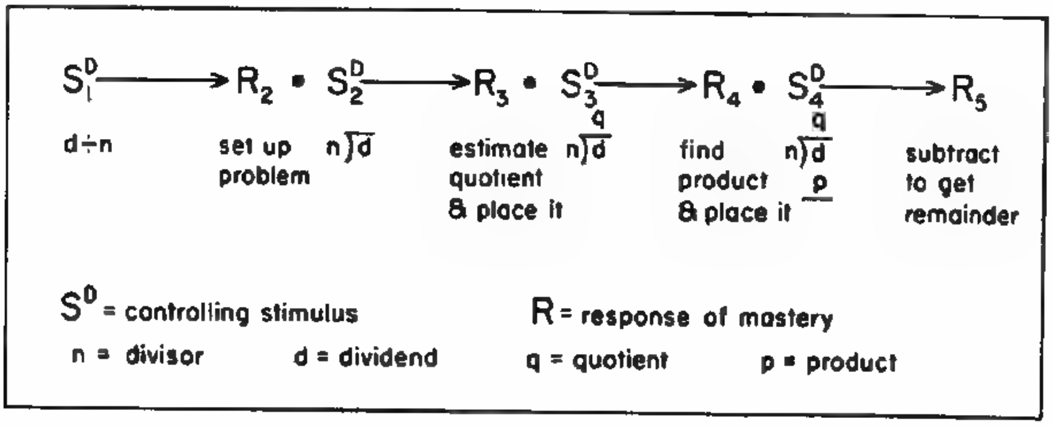



Figure 1. Long division as a behavior chain.
(Simplified for this example)

The principle of chaining tells us that we should first teach the student the terminal operant in the division chain: subtracting the product of quotient and divisor to obtain a remainder. The exercises in Figure 2 are simplified to illustrate the basic plan of all mathetical exercises; in actual lessons, the child would face an exercise looking very much like these, but prior instruction of the type described in a later

**1. Divide 45 by 11**

*Here is what you do:* a) Since 4 x 11 is 44, the 44 is placed under the dividend

$$\begin{array}{r} 4 \\ 11\overline{)45} \\ \underline{44} \end{array}$$

*Now complete the long division:*

b) Subtract 44 from 45 to get the remainder

**2. Divide 28 by 12**

*Here is what you do:* a) 12 goes into 28 2 whole times

b) Multiply the divisor by the quotient (12 x 2) and put the product under the dividend

$$\begin{array}{r} 2 \\ 12\overline{)28} \end{array}$$

c) Subtract to get the remainder

**3. Divide 33 by 15**

a) 15 goes into 33 2 whole times

b) Put the 2 in place above the line

c) Multiply the divisor by the quotient and put the product in its place

$$15\overline{)33}$$

d) Complete the division

Figure 2. Exercises in long division (simplified) designed on the mathetical exercise model

section would be an important determinant of his success. In Figure 2, exercise #1 establishes the terminal operant of the chain described in Figure 1. Exercise #2 establishes the next to the last operant, and the operant before this is established in exercise #3. The entire model for exercises can be shown by these three exercises. An analysis of exercise #1 will reveal the model. In this exercise we are establishing the operant $S^D_4 \longrightarrow R_5$ (subtracting to get the remainder). To establish this or any operant, three things are essential:

a) The animal must be stimulated by $S^D$ (he must attend to the appropriate stimulus or it is not really a stimulus at all).
b) The student must make the response while he is under the influence of $S^D$.
c) Immediately upon making the response, the student must see a result that is reinforcing to him.

In exercise #1 we have arranged these three conditions for the operant, $S^D_4 \longrightarrow R_5$.

First: We arrange for the student to observe the $S^D$ (product in place under dividend). We do this by stimulating and observing response: ($S^O \longrightarrow R^O$). In the exercise, part (a) and the solid arrow together constitute the observing stimulus ($S^O$).

Second: Some stimulus existing at strength for the response is used to get the response made. Usually, a simple verbal instruction ($S^I$) is an adequate stimulus for this purpose. In exercise #1, part (b) and the broken arrow serve as an ($S^I$).

Third: The reinforcement factor should be inherent in the results of mastery performance itself; from the terminal operant comes the end-product of mastery. It is necessary that the student be able to recognize this end-product. It is assumed that the achievement of this end-product is reinforcing to the student, and it will be if mastery is an educational objective of the student.

The basic exercise model for introducing a single operant can be schematized as:

$$(S^O \rightarrow R^O) \cdot S^D \longrightarrow (S^I) \longrightarrow R.$$

The $S^O$ is a stimulus for an observing response, designed to get the student to observe the $S^D$. $S^I$ is a stimulus designed to produce the R, and it is usually a simple verbal instruction.

The observing stimulus ($S^O$) in the exercise model plays two roles, each representing fundamental properties of perception. $S^O$ directs the student's attention to $S^D$, leading him to locate its physical topography. But locating the physical topography is not sufficient to establish the $S^D$; its identity, too, must be established. The number *44* in exercise #1 is a physical form that can, and does, perform many services in the experience of the student; it can have many identities. We do not want to establish $R_n$ on the occasion of any even number or any multiple of 11, but we want it to come under the control of any product of the divisor and quotient. Observing $S^D$, therefore, means not only to locate the *44*, but to recognize its identity in this particular context. The two components of the $S^O$ are, then, an attention-directing element and an identifying element, symbolized $S^a$ and $S^c$, respectively. In exercise #1 of the example $S^a$ functions through the solid arrow and the statement, "the 44 is placed under the dividend." The function of $S^c$ is carried by "4 x 11 = 44" and the student's observation that 4 and 11 are the quotient and the dividend.

In exercise #2 of Figure 2, the next-to-last operant ($S_3 \longrightarrow R_4$) in the chain of long division mastery is treated as the last operant was in exercise #1. The reader should be able to locate the $S^O$ and $S^I$ for this operant. In all essential respects exercise #2 is the same as #1, except that #2 also deals with the last operant. After $S_3 \longrightarrow R_4$ is introduced, the exercise prompts the student to make the next response in the chain. This prompting stimulus, $S^p$, is the statement, "subtract to get the remainder." The purpose of this prompt is two-fold. As a verbal stimulus, it is simpler and more general than the $S^I$ in the previous exercise; it is a stimulus that the student can later give to himself until the $S^D$ is established at full strength as an occasion for the response. This is the *mediating* function of $S^p$. The second and obvious purpose of $S^p$ is to make certain that the terminal operant occurs immediately

after the preceding operant, since it is a behavior sequence that we are trying to establish. This is the *chaining* function of $S^p$.

It is not necessary to provide some residual of the $S^o$ for the terminal operant in the second exercise, because the terminal $S^D$ is created by the student's own response and this assures that he will observe the $S^D$.

Exercise #3 repeats the pattern established in #1 and #2. The third operant from the end is established by an $S^o$ and an $S^I$ and the next operant is prompted ($S^p$). This time, however, the last operant is not guided at all, but the student is released to perform on his own by some statement such as "complete the division." All the exercises to follow would be designed according to this plan; a new operant is introduced, the operant that follows is prompted, the next operant is released, and the remainder are performed as a natural chain of mastery. A generalized lesson plan summarizing the use of the model is given in Figures 3 and 4.

The reinforcement for the terminal operant was the end-product of mastery. The reinforcement for the operant preceding the terminus of the chain is the final $S^D$. If the consequence of the last response is reinforcing, the occasion for this response will be reinforcing, and any operant which produces that occasion will be strengthened. This is a basic principle of behavior theory, and from it strictly follows the principle of chaining. Stated another way, an otherwise neutral event will become valued when it becomes the occasion for something else of value. One of the distinctive features of mathetics derives from this principle. Emphasis of the importance of providing immediate reinforcement in teaching materials is the primary basis for the movement called programmed education. In a program, when a student makes a response he is guided by one of many methods to see the correct response represented by the writers. The ability to check the correctness of his response after he makes it is assumed to be the immediate reinforcer. A mathetical lesson differs from this programming approach in two ways. First, the reinforcer is assumed to be given directly by the product of the student's own performance and not through the medium of an answer sheet; this well-tested assumption is a reflection of the principle of conditioned reinforcement described above: each response produces the conditions for behavior that the student

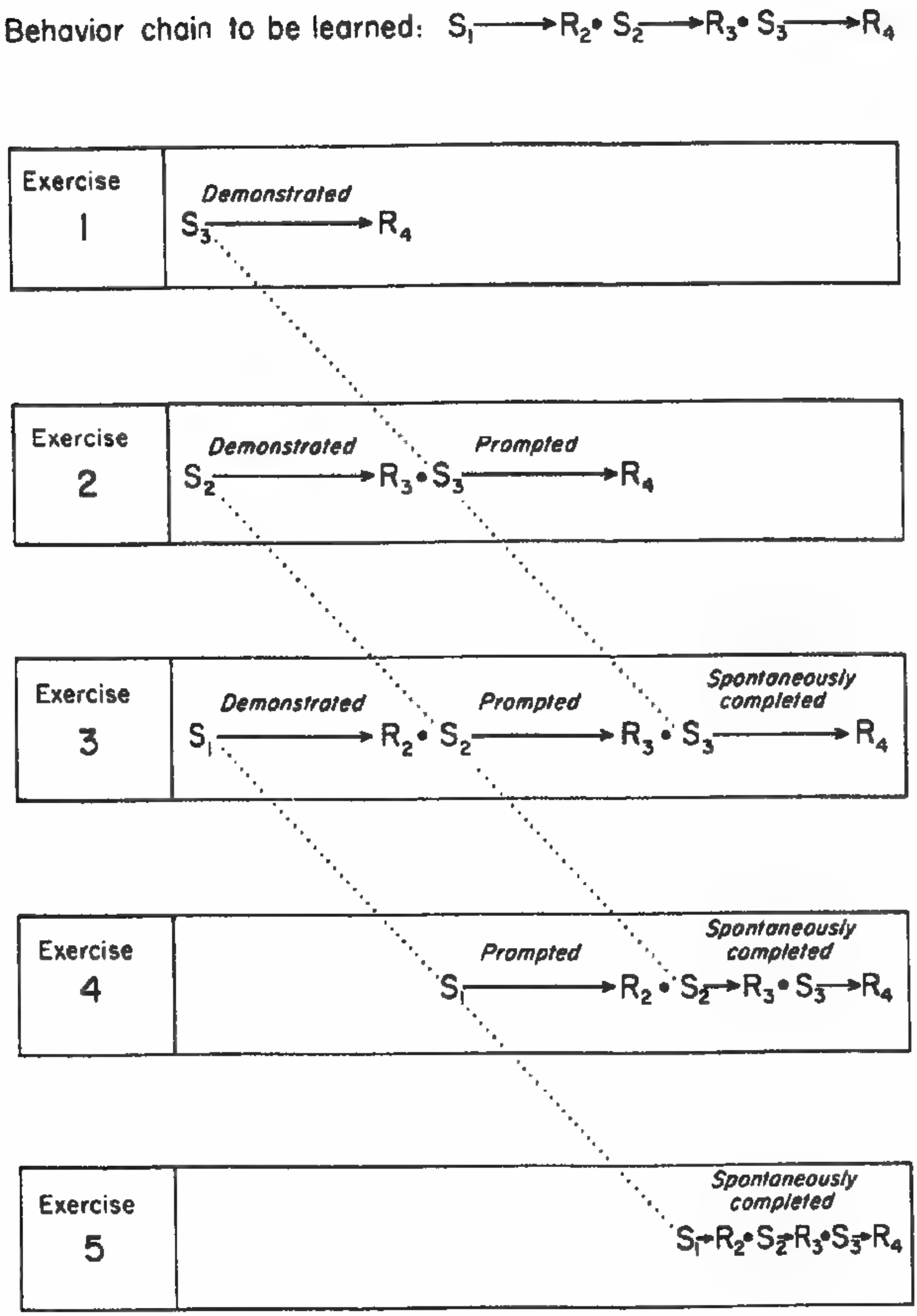



Figure 3. Skeleton of mathetical lesson plan. Beginning with terminal operant, each operant is (1) introduced with full demonstration, (2) prompted in the next exercise, (3) performed spontaneously thereafter.

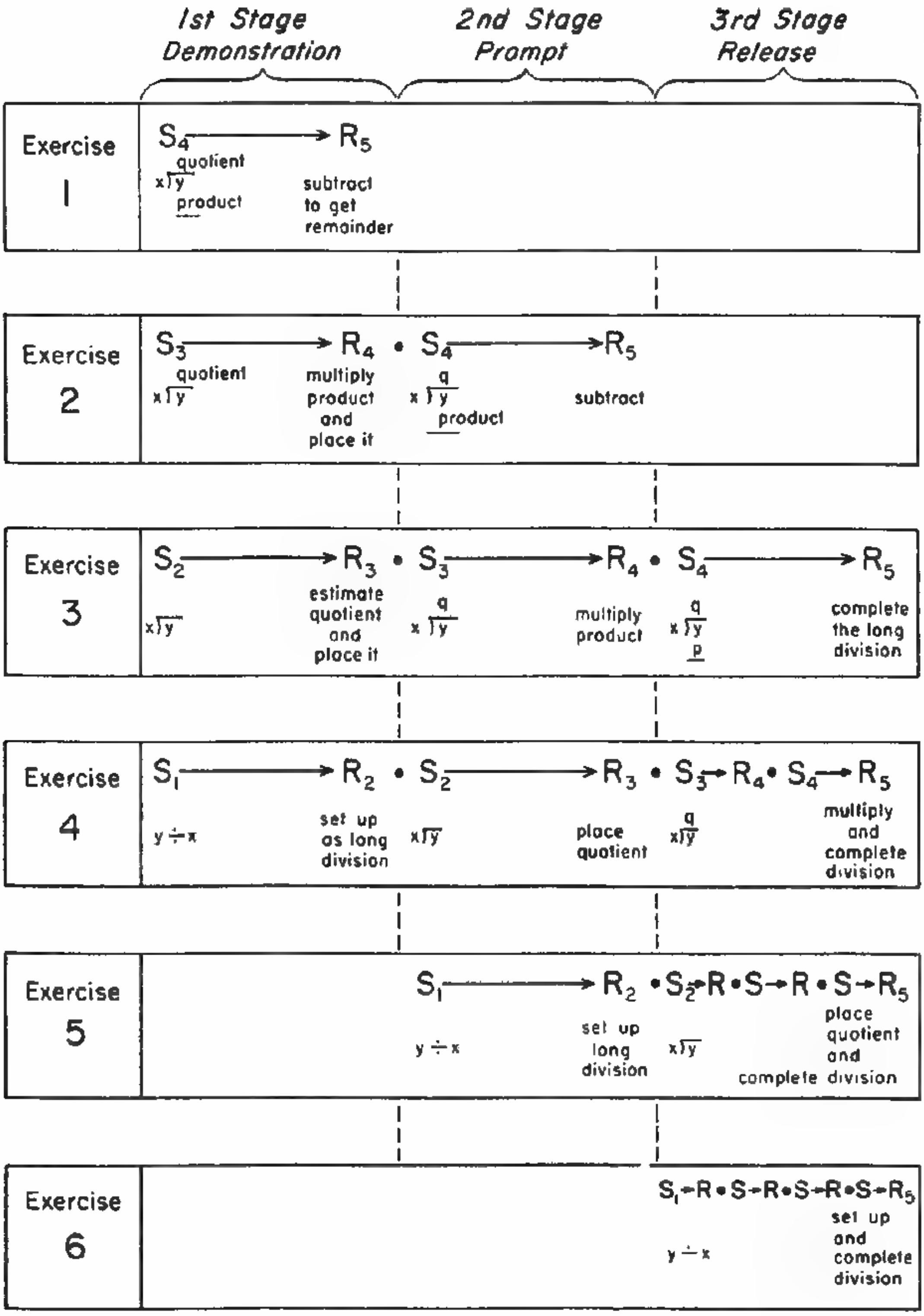



Figure 4. Schematic of lesson plan for long division, indicating the three stages of each exercise in developing a behavior chain.

knows how to perform, and this continues until the mastery sequence is complete. In theory, it is not necessary to give the students any answer checks[1]. Second, the reinforcer in a mathetical lesson is truly immediate, being simultaneous with performance itself, and not delayed by answer seeking. Moreover, the student can minimize his reliance on someone else's word that he is correct; he has seen the end-product of mastery, and knows that his performance is leading there again.

*The Size of an Exercise*

An exercise is a technical unit of mathetics, and it should not be confused with a "frame." A frame is a physical unit that refers to the space in which materials are exposed to the student. An exercise is a behavior unit, and it is defined as all the material designed to establish a single new operant in the chain of mastery. A single exercise may employ several frames, or several exercises may appear in one frame, depending upon factors not basic to our consideration. Stated another way, an exercise is designed to produce a unit of behavior change, one kind of behavior change may require more physical material than an equal behavior change of another kind.

*The operant span* is the basic element of behavior change in mathetics, and in behavior terms the span is a constant unit independent of both content and the repertory of the student. The operant span is the largest gain toward mastery that can be produced in a single exercise.

An understanding of the concept of the span requires an understanding of the arbitrary nature of all scientific elements. Various practices of physical science lead to different choices of physical elements, according to the convenience and purposes of the specialty. The unit element chosen may vary from atoms to molecules, from rocks to drops; always, however, the elements have in common the properties fundamental to all physical particles; all can be described in terms of mass, time, and space. Just as the elemental physical particle is chosen arbitrarily, in accordance with the physicist's purposes, the properties defining a behavior element are arbitrarily set by the behavior scientist to be consistent with his aims. We can treat each single arithmetic operation as a single

[1] What is true in practice will be described later in the series.

operant, or we can view the whole process of long division as a single operant. Nothing restricts us, for any observable operant can be analyzed into a sequence of multiple operants, or combined with other operants into one. Nothing restricts us, that is, but our purpose.

Now, the purpose of the matheticist is to create materials that will produce the greatest behavior change at the least cost (however these costs may be accounted). Consistent with the purpose of mathetics, the basic unit of behavior is defined as *the greatest possible behavior change that can be produced by a standard effort.* The mathetical exercise model represents the standard effort, and the operant span is the maximal change in the direction of mastery made possible by that exercise. It follows that the ideal exercise will include as much performance as embraces the operant span. The principle for determining the size of the exercise is *not* "break the materials into small parts"; the principle is to require in every exercise as much mastery performance as the student can reasonably negotiate.

*Determining the span.* The operant span is first estimated in an early stage of mathetical analysis, during the description of mastery behavior; an empirical evaluation of the estimates occurs in the lesson try-out stage. Details of these procedures are described elsewhere. Generally, however, one can think of the span as embracing all the performance that a student could carry out if he were following a set of instructions. If you gave him the contents of an exercise as a set of instructions, he could fail to perform adequately for three reasons:

(a) The materials designed to function as $S^O$ and $S^I$ are not working properly.
(b) The amount of work required of the student is too great for the reinforcers he is getting.
(c) The operant span has been exceeded—the performance was too involved for him to remain under the influence of $S^O$ while attending to $S^I$.

The span as a unit assumes the reinforcement value of accomplishment and the adequacy of $S^O$ and $S^I$. In the try-outs of the material the try-out editor knows the function of every part of every exercise; when a student fails to negotiate the

exercise, the experienced editor can usually trace the failure to one of the functions above.

*Complex Exercises*

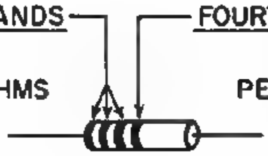
We have considered until now the teaching of simple behavior chains in which all the component operants exist at considerable strength; the problem has been to establish a fixed sequence of familiar operants. The problems of learning are not, however, restricted to establishment of behavior chains. In many subjects the chains that the student must learn are trivial behavior problems that he could accomplish rapidly if he had mastered all else.

In long division, the student's greatest difficulty usually lies in remembering the proper sequence; he has already learned to discriminate numbers from letters, to perform multiplication, etc. In contract law, however, the sequence the student must learn to go through to interpret a contract is, by itself, no challenge. His real problems exist with just a few operants having stimulus components that he has great trouble generalizing; he has to learn to make the same response to a stimulus that occurs in many varied forms to which he has learned to make opposing responses in the past. In learning the color code for electrical resistors, in which he must read colors as numbers, the student has no trouble in generalizing the color stimuli, and the chain of behavior he must learn is simple. Here, he has to learn to make 10 different but highly competing responses to 10 different colors; a new discrimination offers the real learning problem. The primary contributions of a mathetical analysis are: (a) the distinction of behavior difficulties peculiar to a given subject matter, and (b) the rules for teaching strategies that are best suited to these several difficulties. The exercise model is, however, basic to all the behavior problems. To illustrate the generality of the model, Figure 5 presents the three exercises from an electrical resistor lesson that teach the number each of the 10 colors stands for.

This example is doubly instructive. First, it illustrates the teaching of a multiple of discriminations in the same three-stage process of demonstrating, prompting, and releasing that is applied to a single discrimination. Second, it demonstrates the variety of forms in which the exercise components can occur.

1a. Some electrical resistors have COLOR BANDS that tell how much they will resist electric current. On small resistors you can see colors better than numbers. Each color stands for a number.

THE FIRST THREE COLOR BANDS ARE READ AS THE NUMBER OF OHMS RESISTANCE



THE FOURTH COLOR BAND IS READ AS THE PERCENT OF ERROR IN THE RATING

1b. Each of the FIRST THREE COLOR BANDS can have one of 10 colors. Read through this list twice. Learn the NUMBER for which each COLOR stands.

a FIVE dollar bill is GREEN
ONE BROWN penny
a WHITE cat has NINE lives
SEVEN PURPLE seas
a BLUE tail fly has SIX legs
ZERO: BLACK nothingness
a RED heart has TWO parts
THREE ORANGEs
a FOUR legged YELLOW dog
an EIGHTy year old man has GRAY hair

2. List the number for which each COLOR stands:

| | | | | |
|---|---|---|---|---|
| RED____ (heart) | WHITE____ (cat) | PURPLE____ (seas) | BROWN ____ (penny) | BLACK____ (nothingness) |
| GREEN____ (bill) | GRAY____ (hair) | BLUE____ (tail fly) | ORANGE____ (oranges) | YELLOW____ (dog) |

3. List the NUMBER for which each color stands:

| | | | | |
|---|---|---|---|---|
| BLACK____ | BROWN____ | YELLOW____ | GRAY____ | GREEN____ |
| WHITE____ | PURPLE____ | RED____ | ORANGE____ | BLUE____ |

Figure 5. Sample exercises. From lesson on color codes of electrical resistors.

Several parts of the first exercise serve the functions of $S^O$, including the capitalization and underlining. The $S^I$ functions largely through the sentence, "Read through this list twice," but also through the typography and general layout. The response stimulated occurs covertly as the student reads —a fact that would seem too obvious to need mention, if it were not true that many writings on programmed education seem to take the word *response* to mean an overt act with a pencil. All the properties of the exercise model are here; the only real addition are the associative aids like *penny, seas,* etc.; the function of these will be discussed later. These additions, however, recur in exercise #2 in a familiar role; they serve as the $S^p$. In exercise #3 the operant is released.

*Exercise formats* in mathetical design are determined only by the requirements of the lesson; no fixed form is, or can be, advocated in advance. The exercise model should not suggest a fixed form; the components of the model refer to *behavior functions* that must be served, not to spatial arrangements. If context and well established conventions insure that these functions will occur, it is wasteful to supply the $S^O$ and $S^I$ with an arbitrary physical form.

## PART III. PRESCRIBING A MASTERY REPERTORY

We may easily agree that the design of an educational sequence properly begins with a description of the behavior it is intended to create. It is not as easy to agree how to render the description or what we shall include. We have to decide what behavior is intended by the names usually applied to knowledge domains, and seldom will the experts be found in full agreement. Also, we need standards of precision and detail, for even the pettiest tasks can be analyzed into many acts, while skills of the highest order and complexity can be summarized in a few words. If our descriptive system is to meet the exacting requirements of technology, we will need something more than guess-work to guide us in these decisions.

Mathetics contains a systematic technique for detailing the explicit behavior objectives of instructional design. The first formal procedure in mathetics is the development of a behavior *prescription.* This word emphasizes both the provisional nature of the task description and the necessity for

deciding what behavior constitutes mastery. There are no fixed domains of knowledge, only behavior complexes prescribed by the public as educational objectives.

*Nature of a Prescription*

The mathetical prescription is a description of the behavior of a master of some knowledge domain, and is sufficiently precise that a master reading it should have no doubt about the objectives of any course built upon it. A prescription symbolizes a complex of mastery behavior in units of the *operant span.* Each operant represents an estimate of the largest gain toward mastery that could be effected in a target student by a single exercise. The operant span is the descriptive unit determined for the purpose of the lesson; what is really being described is the behavior that must be established to carry a student from his initial repertory to a repertory of mastery.

The prescription is written within the framework of stimulus-response notation. The importance of this notation lies partly in the ease with which it allows the matheticist to distinguish between the stimuli and the responses of mastery, and partly in the varieties of behavior properties that are immediately revealed by the notational system.

The basic determinants of optimal teaching sequences are not found in the content of the domains of knowledge, but they are seen in the particular stimulus-response structure of the prescription. A simple chain of behavior is best established by retrogression through the basic exercise model, whether the chain represent long division, molding dental braces to the proper shape, or balancing a teller cage in a bank. A multiple discrimination requires tactics additional to the exercise model, whether the discrimination involves resistor color bands, chemical reactions, or situations posed by a customer in a department store. Generalizations require a set of tactics independent of subject-matter content. Prescription notation is a convenient way to depict the behavior structure of a domain without the confusion of content.

Basically, there are three kinds of behavior structures: chains, multiple discriminations (called *multiples*), and generalizations. They can be represented in S-R notation as:

(*Chain*) $S \longrightarrow R \cdot S \longrightarrow R \cdot S \longrightarrow R \cdot S - \ldots\ldots \rightarrow R$

(*Multiple*)
$$\begin{array}{l} S \longrightarrow R \\ S \longrightarrow R \\ S \longrightarrow R \end{array}$$

(*Generalization*)
$$\left.\begin{array}{l} S \\ S \\ S \end{array}\right] R$$

Typically, one of these structures tends to be the dominant characteristic in the prescription of a particular subject-matter, although the structures can combine and repeat themselves in unlimited ways. Figure 6 is the prescription for reading color coded electrical resistors; all three structures are represented in this behavior. In Figure 6, the several properties of the prescription can be seen. Each operant represents an estimate of the operant span. All the behaviors are sufficiently explicit that a master of the subject can understand exactly what is described. The dominance of discrimination as the chief learning problem in color coding is readily obvious. The operants do not represent verbal behavior about mastery, but each operant is a true element in the synthesis of mastery.

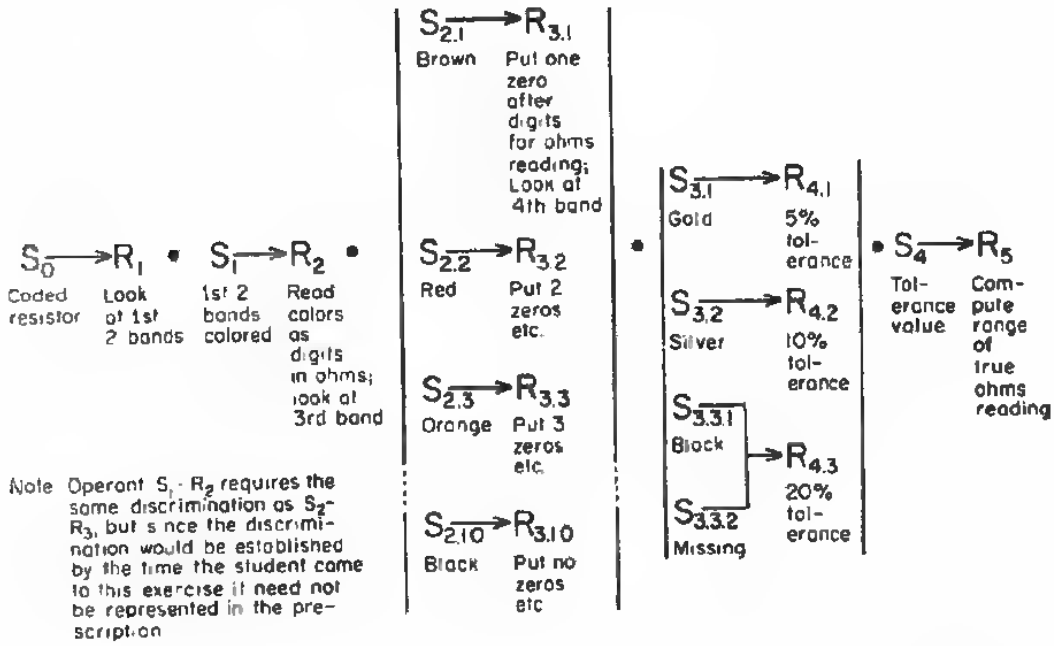



Figure 6. Prescription for reading color-coded electrical resistors.

*Writing a Prescription for a Synthetic Repertory*

A detailed account of prescription writing will fill a separate treatise later in this series. The fact is that, depending upon the subject-matter, prescriptions are either quite simple to write or made unusually difficult by lack of experience in observing certain kinds of behavior. Many of the detailed rules we employ for writing prescriptions are designed to protect matheticists from their own biases in observing behavior.

A prescription is developed in a series of approximations. The first approximation (1st Px) describes the step by step synthesis of mastery in operant units considerably smaller than the operant span. The use of small units in the 1st Px provides a safeguard against the omission of any necessary behavior details; this prescription will be checked by the person responsible for specifying the educational objectives, and many potential misunderstandings can be avoided at this stage. The 1st Px is not a reliable picture of the dominant behavior structures, since they are usually hidden in the unnecessary detail.

The 1st Px may be written with or without the help of an expert in the subject-matter. Little help will be needed from an expert at this stage unless the subject is very advanced or highly specialized. Expert help usually comes during an earlier, preparatory task survey, or later when the first exercises are written.

The second approximation of the prescription is developed from the 1st Px. The matheticist begins, for the first time, to estimate the operant span. He does this by examining a sequence of operants and asking if the target student could perform the sequence as a single act after reading or hearing instructions telling him what to do. This combining of several operants into one continues until it appears certain that a wider combination will exceed the span or until the estimate becomes uncertain. Uncertainty about two operants exceeding the span if combined is resolved by a practical mathetical rule: When in doubt, combine them; always let your biases be in the direction of assuming too much about the student's repertory. This rule is determined by the simple fact that if you overestimate the student's repertory, you will discover the error instantly when the lessons are first tried out. If you err in the other direction, it will be very difficult to discover

it. Moreover an error of overestimation can always be rectified with little effort.

After completing the 2nd Px, the matheticist re-examines his estimates, and not infrequently he will arrive at a third approximation. The complexity of the domain and experience will determine the number of approximations necessary to produce a confident prescription; this approximation is called the Nth approximation (Nth Px), not the final Px, since more is to be done. Figures 7a and 7b illustrate the development of a prescription through three approximation stages. The subject-matter is the use of a slide rule for multiplication. The final operants represent chains that recur repeatedly in all manipulations of the slide rule: reading a scale and determining decimal positions. These operants are analyzed in another prescription. This example is taken intact from the work of a new trainee in mathetics.

### *Tactics Analysis*

The Nth Px does not necessarily represent the most efficient possible mastery performance, even if it unequivocally reflects the behavior of all experts. More efficient performance often can be discovered and centuries of perfect agreement among experts is no guarantee to the contrary. The rigorous scrutiny that mathetics applies to mastery behavior often reveals inefficiencies in accepted practices. A systematic procedure for assessing the tactics of mastery has been incorporated in mathetics. The procedure is:

(a) The tactics analysis begins with an examination of each operant in the Nth Px. The operant is listed in a vertical column and the matheticist describes in an adjacent column every alternative operant that could be substituted without changing the object of the total sequence. This description continues for every operant in the prescription.

(b) Each alternative (if there be any) is examined for its mechanical efficiency compared with the prescribed operant, and it is designated by a simple notation as more, less or equally efficient.

(c) Alternatives noted as more efficient are next examined for the effects of their substitution on

> other operations in the prescription. An operant somewhat more efficient than its alternative, when they are compared in isolation, may require later operations that could be avoided if the alternative were substituted. What is sought is the most efficient chain.

The use of the C and D scales of a slide rule will illustrate the results of a tactics analysis. In actual operation, experts will begin the multiplication by moving the slider bar from left to right in order to place the left index of the C scale over the multiplier on the D scale. Since the scales of a slide rule are logarithmic, most of the numbers appear on the right-hand half of the scales. If the left index is used the product will not appear in register with the C scale eighty-two percent of the time, and the slider bar must be moved back to the right. If the user began with the right index, he would have to reset the scale only eighteen percent of the time. This apparently unknown inefficiency in the use of the slide rule is routinely discovered in the course of a tactics analysis. The matheticist examines the operant description that reads, "move the left index to the right until it is in register. . . . ," and he sets down all alternatives, only one of which he can discover. This is an especially good example of how a tactics analysis can lead to improvements over well defined and universally accepted practices although the particular achievement is a trivial one since the C and D scales are used for multiplying only on very cheap slide rules.

*Analysis of Individual Differences*

Apparently, individual differences exist for the sole purpose of confusing educational issues; there has even developed a "science" of individual differences, in spite of the fact that the very notion is contrary to the nature of science. All science has as its subject-matter individual differences; science is the grand attempt to rise above them to higher abstractions of common properties. In education, individual differences among behavior repertories are unusually difficult to discuss with detachment, simply because they are confused with the value we place upon human individuality. The issue is much simpler if we agree that an educational technology is applica-

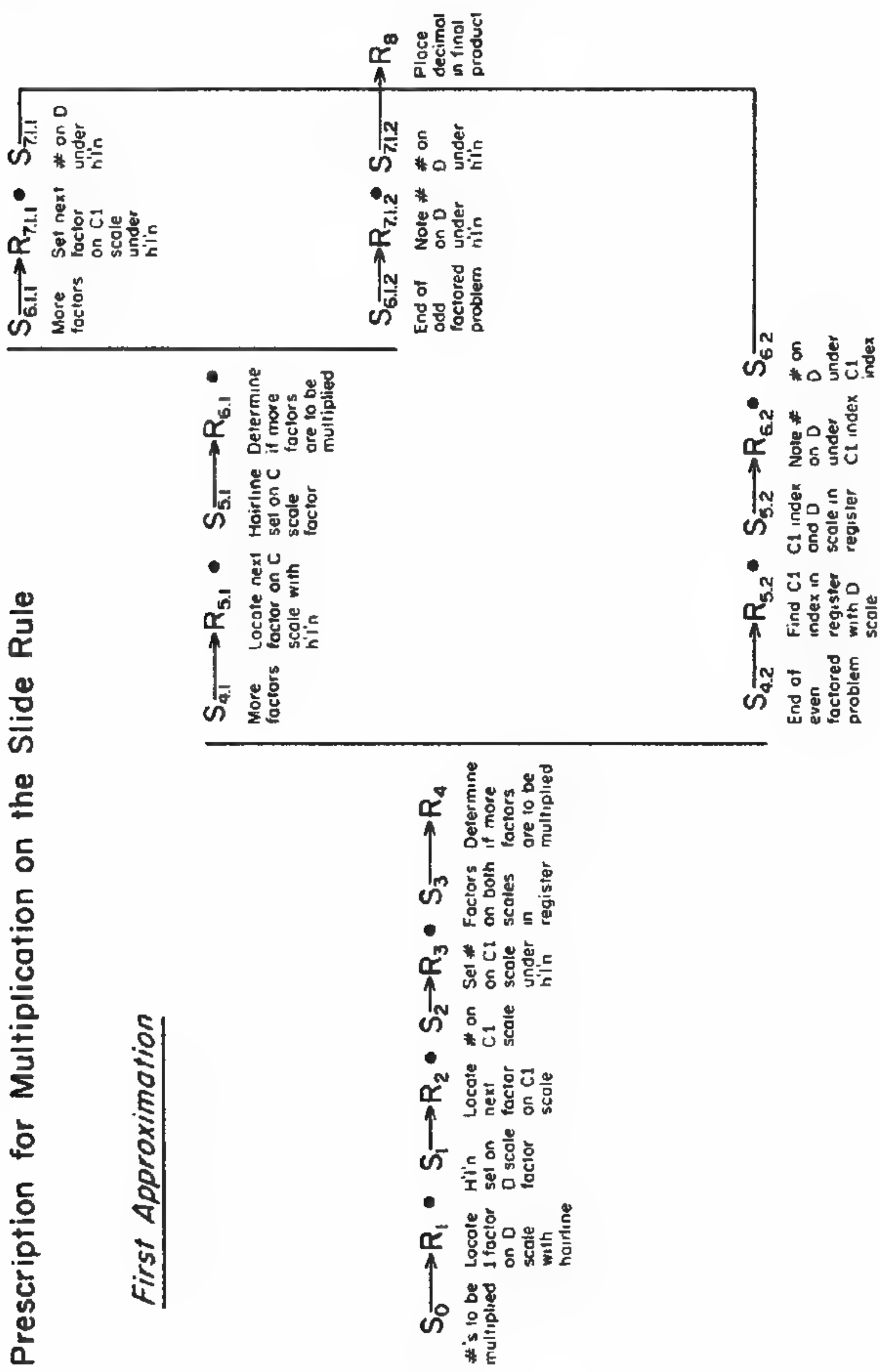

Prescription for Multiplication on the Slide Rule
First Approximation
S0 → R1 • S1 → R2 • S2 → R3 • S3 → R4
#'s to be multiplied
Locate 1 factor on D scale with hairline
H'l'n set on D scale factor
Locate next factor on C1 scale
# on C1 scale
Set # on C1 scale under h'l'n
Factors on both scales in register
Determine if more factors are to be multiplied
S4.1 → R5.1 • S5.1 → R6.1 •
More factors
Locate next factor on C scale with h'l'n
Hairline set on C scale factor
Determine if more factors are to be multiplied
S6.1.1 → R7.1.1 • S7.1.1
More factors
Set next factor on C1 scale under h'l'n
# on D under h'l'n
S6.1.2 → R7.1.2 • S7.1.2 → R8
End of add factored problem
Note # on D under h'l'n
# on D under h'l'n
Place decimal in final product
S4.2 → R5.2 • S5.2 → R6.2 • S6.2
End of even factored problem
Find C1 index in register with D scale
C1 index and D scale in register
Note # on D under C1 index
# on D under C1 index

Figure 7a

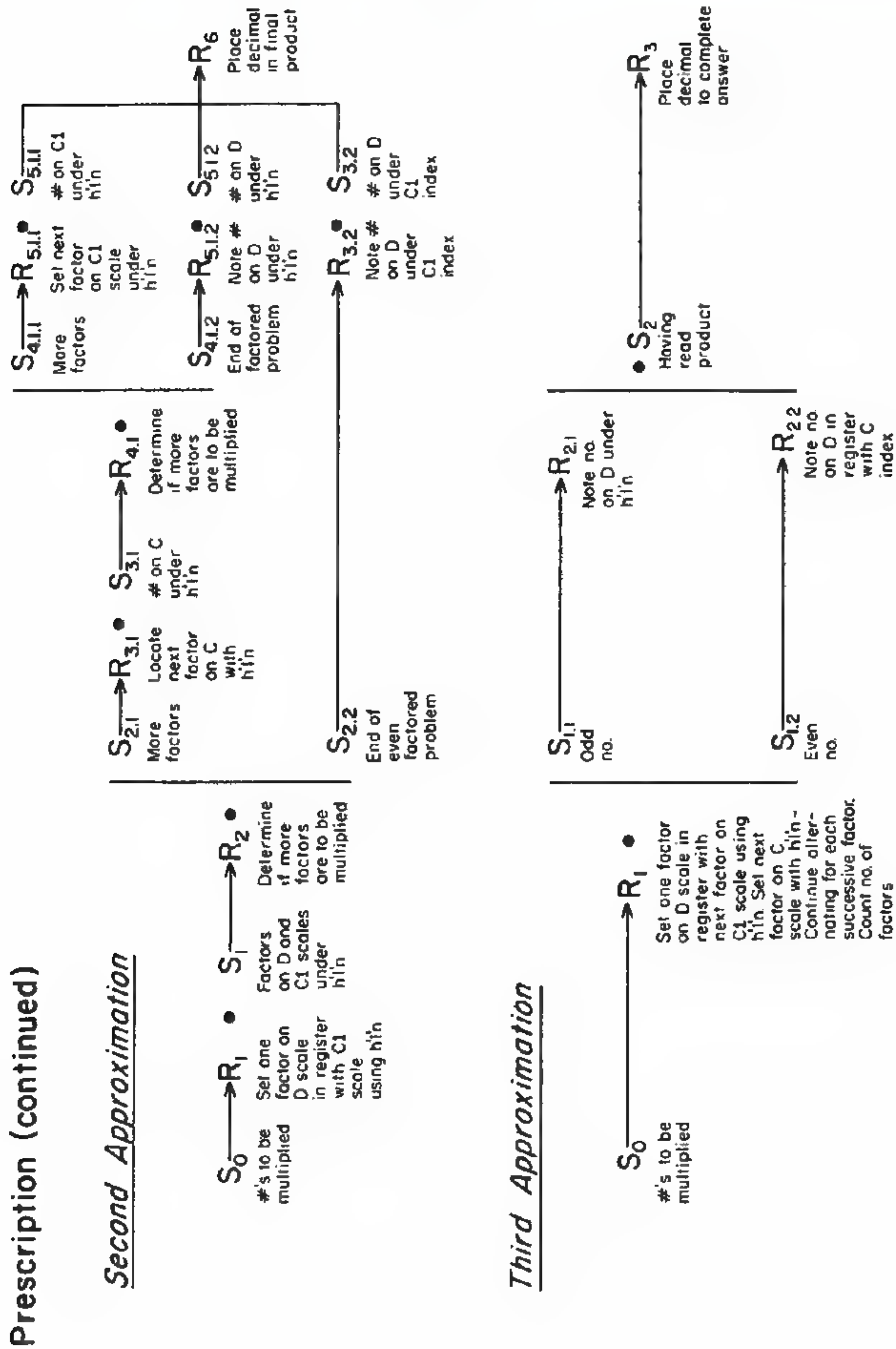

Prescription (continued)
Second Approximation
S0 → R1 • S1 → R2 •
#'s to be multiplied
Set one factor on D scale in register with C1 scale using h'l'n
Factors on D and C1 scales under h'l'n
Determine if more factors are to be multiplied
S2.1 → R3.1 • S3.1 → R4.1 •
More factors
Locate next factor on C with h'l'n
# on C under h'l'n
Determine if more factors are to be multiplied
S4.1.1 → R5.1.1 • S5.1.1
More factors
Set next factor on C1 scale under h'l'n
# on C1 under h'l'n
S4.1.2 → R5.1.2 • S5.1.2 → R6
End of factored problem
Note # on D under h'l'n
# on D under h'l'n
Place decimal in final product
S2.2 → R3.2 • S3.2
End of even factored problem
Note # on D under C1 index
# on D under C1 index
Third Approximation
S0 → R1 •
#'s to be multiplied
Set one factor on D scale in register with next factor on C1 scale using h'l'n. Set next factor on C, scale with h'l'n. Continue alternating for each successive factor. Count no. of factors
S1.1 → R2.1
Odd no.
Note no. on D under h'l'n
S1.2 → R2.2
Even no.
Note no. on D in register with C index
• S2 → R3
Having read product
Place decimal to complete answer

Figure 7b

ble only to animal behavior, that man is at least an animal, and that differences among men's animal skills could be totally obliterated without affecting the differences in the way they chose to use them. It seems to me that much of the fear that individuality will be destroyed if we teach all men the same skills arises from an uncritical equation of the ability to perform and the decision to perform. Art schools have more and more avoided teaching traditional skills, presumably to protect the student's creativity, which I take to mean his decision to use his skills in unusual combinations.

Mathetics is a specialized animal training system adapted to the human animal; its techniques do not extend with any authority to the problems of human value. The success of any mathetical lesson is dependent upon the correctness of the assumption that there are students who value the mastery made possible by the lesson. The purpose of mathetics is to create efficient routes to this mastery.

Efficiency has polarity where individual differences exist. Lessons individually tailored for each student might cost greatly in the efforts to produce them; a single lesson designed for the average student will create failure and unnecessary efforts. Optimal strategy would appear to lie somewhere between the two; however, the appearance is misleading and based on a fundamentally unsound assessment of the extent of individual differences. Mathetical analysis makes possible individualized lessons in advance of knowing who the individuals are, and at the same time keeps the bulk of the lessons greatly reduced.

The assessment of individual differences traditionally confuses components of a repertory with the effects of the repertory. I say I know nothing about marksmanship, that the difference between my skill and the sharpshooter's is enormous. But is it not true that I know how to lift a rifle, find the sight, find the trigger, place the correct finger on the trigger, point the barrel at the target, cock the hammer, etc.? There is not a single act in the sharpshooter's repertory that I can not approximate very closely. But a miss is as good as a mile: the value of my repertory falls far short of the value of the sharpshooter's.

The trained student who has mastered arithmetic up to the threshold of long division will make zero on the division test in which his older brother scores 100; yet there is barely a

step in long division which that younger boy has not mastered; he merely has not learned a simple sequential order. The test, we should observe, is usually scored by the right answer—by the effect of the repertory. A small change in a behavior repertory can have a large effect in the environment. The feeling that individual differences are hopelessly great is produced by experiences with variation in the effect of repertories; we seldom even observe the relevant components of repertories that are not producing the desired effects; we simply say, "he doesn't know anything about this."

### *Assessing an Initial Repertory*

The prescription is constructed in units of the operant span, and the operant span must be estimated. Success in making good estimates is much greater than would be imagined; the truth is that any group of people slated for training in the same subject-matter will possess repertories that are strikingly similar in their components, though not in their effects. The difference between the initial repertories and the mastery repertory will, itself, be a small proportion of the total behavior components of either. The estimates are made by observing a target student's existing repertory—not directly, except in rare cases, but vicariously, from memory, or by observing one's own behavior. Deviations of the estimates from true performance will be small and easily corrected.

### *Differential Analysis*

Individual differences are provided for through the following procedure:

(a) We first ask if there are any target students who could not negotiate an operant in the Nth Px for reasons other than the span being exceeded. The only other reason that failure could occur would be that some skills are incorrectly assumed at strength in the prescribed operant. If some target students are known not to possess the component skill, this operant is further analyzed, and a prescription is written for the component, called a *differential prescription* (D-Px).

(b) The D-Px is treated as any other prescription. The exercises written for it are inserted in the lesson at an appropriate place. Each set of differential exercises (D-Ex) is preceded by a diagnostic test item. The diagnostic item is written so that the only reasonable cause of failure is in the lack of skill which the D-Ex is designed to teach. Students who fail the item are directed to the D-Ex.

## PART IV. PRESCRIPTION OF THE ANALYTIC REPERTORY

The prescription of the synthetic behaviors consists of all those overt acts characterizing mastery. A machine designed to perform these acts need never be replaced as long as the requirements of mastery do not change. But these requirements do change, and small alterations in the logic of procedure can represent great changes in performance mechanics. Although the human is remarkably adaptable to change, he, too, can be made obsolete if he lacks a true understanding of the skills he achieves. The great value of human performance rests largely on the human repertory of analytic behavior. The distinction between synthetic and analytic repertories, as these terms are used here, is parallel to the distinctions implied by contrasting words like "theory" vs. "practice," or "doing" vs. "understanding."

Imagine a household electrician who is able to use a selenium rectifier (a device used to change AC current to DC). We give him a radio tube and tell him that it is a rectifier: no two electrical instruments could look less alike than a tube and a selenium rectifier, and he cannot see their essential similarity. To train him to use the tube, we must begin at an elementary level. Compare this electrician with a physicist who is thoroughly grounded in the theory of rectification, but who has never seen a selenium rectifier; initially, the physicist is as baffled as the electrician was by the radio tube. But here is the difference: with one simple word, rectifier, our physicist is transformed from a helpless novice to a near expert. One word renders such magical mutation that it is understandable why we place great value on the teaching

of theory. We also become suspicious of theory when we see a man at a loss before a simple rectifier after he has spent years studying the theory of rectification.

What is in the repertory of the physicist that he can become so rapidly a master of the new? What is missing from the organization of the electrician's repertory that he is incapacitated by a few superficial alterations in a thing he has so thoroughly mastered? We can find the answer only by looking at theory as a behavior repertory.

*The Pedagogical Function of Theory*

The word *theory* is derived from a Greek word meaning "to look at," and its etymology is a guide to a useful way of understanding it. The *Oxford English Dictionary* tells us that the word is used only in a late and loose sense to indicate conjectures and untested hypotheses (the favored use current among psychologists).

Preferred usage emphasizes theory as a view of something to be done, or a method of doing it—an hypothesis that has been confirmed by observation. Let us restate this in behavior terms.

Theory can be thought of as a repertory of selective looking behavior, as a set of words that we use to stimulate ourselves to look at the key and relevant features of an otherwise confusing jumble of detail. We can teach a student to draw a box in perspective from a floor plan without teaching any theory of perspective at all. He will learn to draw a series of lines and points on paper until he has a three dimensional representation; but something is missing and the student knows it. He will likely say, "I can do it, but I don't understand what I am doing," a familiar complaint of students. For practical purposes, what difference does it make that he does not understand if he can do the job? But there is something of substance behind the student's uneasiness. Because we failed to teach him theory, he will surely fail to generalize his skill to other operations of the same essential kind. Ask him to change the lines of construction so that the box he draws will look like a box that is seen from underneath: he can not do it, because he has not learned which of the many points and lines are key to determining the appearance of the box. Knowledge of theory would allow him to select from all

the possible arrangements of lines those that would lead to a prescribed result. If he had gained an organized verbal repertory to stimulate this selective behavior, he would not have to learn anew many of the variations; moreover, he would not readily forget what he had mastered. It is far simpler to retain a simple verbal behavior chain if that chain prompts the overt acts of drawing, and is prompted by those acts.

The teaching of theory has three functions in education:

(1) It fosters generalization by mediating the behavior of *looking* through superficial details to essential properties.
(2) It increases retention by serving as verbal prompts for the overt acts of mastery behavior.
(3) It increases reinforcement by shortening the route to mastery and by relating the behaviors in learning more directly to the objectives of the student.

### *How Much Theory Should Be Taught*

Theory is a behavior repertory and is prescribed on much the same basis as we prescribe the synthetic repertory. Theory is a repertory of verbal behavior. Just a lot of words which—taken in isolation of their effect on other behavior—have no useful result, do not produce a useful product. Theory is useful in mediating, maintaining, and extending mastery. We can assume that those who purchase educational sequences desire to pay only for the amount of theory necessary to produce the generalization required to meet the objectives of mastery. Theory is useful if it can create savings in learning time and improvements in retention to compensate the cost of designing theory instruction.

Suppose a certain industry employs men to operate inspection equipment that yields data to be interpreted through a certain algebraic equation. To decide whether a part is to be accepted or rejected, the inspector reads the data from a dial and makes the appropriate computation. When the company periodically redesigns the part being inspected, a new algebraic formula must be used. The inspectors, we imagine, have learned nothing about algebra except the formulas no longer applicable; therefore, they must be retrained periodically. The

popularly suggested relief from this recurring training problem is to teach courses in high school and college algebra. But why is this relief desirable? If all the future changes foreseen by the company require algebraic formulas representing a particular algebraic function, why should the company stand the expense of teaching any algebra other than this function? Presumably, the company is under no obligation to give a liberal arts education to its personnel unless this is partial pay for services. The extent of theoretical instruction is exactly limited, in a sound economy, to the service of the synthetic repertory prescribed for mastery.

In the public school system, however, the customer and what he pays, mastery and what it returns to the customer, are not readily cost-accounted. How much theory, then, shall the designer of educational materials construct in his lessons? It seems to me that the answer which applies in industry applies here. The responsibility of those who design the teaching materials does not extend to determining the constituents of mastery; technical knowledge of the learning process does not supply special wisdom about what *should* be taught. The matheticist, *as a technical person,* has no alternative but to assume that the repertories of *synthetic* behavior prescribed by the public school authorities represent the best available account of the public's educational objectives. If the matheticist does not assume the authority to establish the curriculum, he can introduce into his teaching sequences only that amount and kind of theory which best serves the learning process; to add further theory is to augment the curriculum without the authority to do so.

Consistent with these considerations is the rule to guide the matheticist in determining how much theory to include in his lessons. *Teach only that theory that is pedagogically applicable to the behaviors prescribed by the curriculum.* This rule does not imply that public school curricula or programs of industrial training represent the most intelligent conclusions about what is good for the educational objectives of the customer; it simply distinguishes the role of the matheticist as a technical person from his role as citizen or stockholder.

There is one area of the curricular adequacy on which the matheticist can speak with technical authority. Curriculum content is determined by two things: one is relevance to the educational objectives; additional materials are included, how-

ever, to help the student learn the curriculum content. The second basis of curricular decision is a technical one, clearly within the scope of the learning specialist. Sixteenth century counterpoint, for example, is a typical inclusion in the college music curriculum, although, until recently, practice of the subject was virtually obsolete. There is no universal agreement as to why emphasis of the subject has continued, but it is frequently defended as necessary for the understanding of modern harmony. The matheticist will properly question any part of the curriculum included to aid the learning process.

*What Theory Should Be Taught*

There exist in abundance theories relevant to some part of any subject-matter. Suppose we are teaching perspective drawing at an elementary level and we seek a theory for our pedagogical purposes. Projective geometry is our most likely candidate, for most principles of perspective drawing can be derived from it. But how do we decide what part of the geometry to choose; and what do we do with those students who are quite ready for drawing lessons, but whose mathematics training has not prepared them for this geometry? Preparation in theory could burden our lessons with far more material than is justified by their purpose.

For every subject-matter there is a theory, or a combination of two or more theories, that could produce the selective understanding desired. There exists in libraries and among experts two general kinds of theories, neither precisely suitable for our purposes. One of these I shall call *realm* theories, the other *special* theories. By realm theories, I mean those more general systems which embrace not only the subject we may be charged with teaching, but much other matter as well. Projective geometry is an example of a realm theory with respect to perspective drawing; accounting is a realm theory for balancing a bank teller's cage; the algebra of natural numbers, for reducing fractions. If we are given the assignment of teaching some prescribed mastery, we shall find realm theories embracing more subject-matter and requiring more background than is necessary for the job. Special theories also exist in profusion as every expert tends to create his own method of systematizing his area; but specific employment in practice leads experts to specialize in some aspect

of their knowledge domains. Realm theories are too inclusive for our purpose, special theories not general enough.

What we seek for pedagogical purposes in any prescribed domain of knowledge or skill is a theory that embraces all the elements of the repertory of synthetic behaviors and no more: a theory uniquely tailored to our assigned subject-matter. I shall call such systems *domain theories*. Unfortunately, domain theories seldom are found ready devised; the delineation of a knowledge domain is, for educational purposes, determined by economic and social factors that vary with time and place. The more general realm theories tend not to reflect these social differences. Mathetical analysis is, therefore, generally faced with developing the theory most appropriate to an assigned domain. In developing a domain theory, the matheticist is frequently able to contribute to a clearer understanding of the subject-matter itself.

### *Developing the Domain Theory*

A domain theory is produced for the purpose of supplying a student with a repertory of selective observing behavior, a repertory that should increase his ability to generalize across the superficial details peculiar to single problems within the domain. This repertory of selective observation is called the analytic repertory. In common terms, the analytic repertory is "theory," the synthetic repertory is "practice." In the learning sequence, the analytic behavior will be taught before the synthetic—the student will learn the theory of a given skill or subject before he learns to synthesize mastery performance. In mathetical development, however, the synthetic behaviors are prescribed before the analytic, and for two reasons. First, analytic components are difficult to isolate without having a detailed synthetic prescription before you. Second, a domain theory, as it is here defined, embraces only the elements of the prescribed synthetic repertory.

The following is the procedure for developing a domain theory. Figures 8a and 8b illustrate these steps.

STEP 1: Rewrite the synthetic prescription, reducing all multiples to a single operant.

STEP 2: Restate the description of each operant in its most generalized and non-substitutable form (by

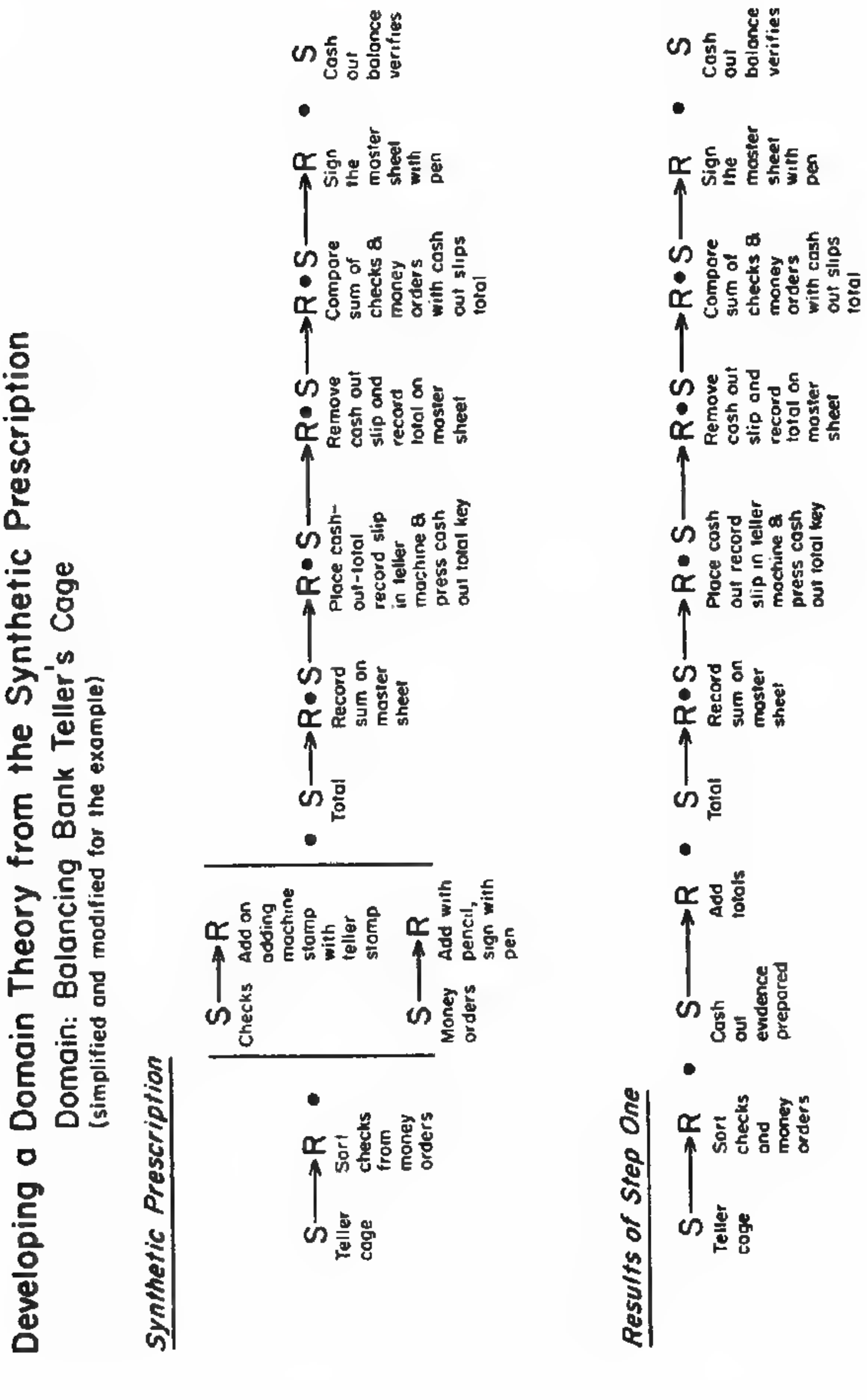

Developing a Domain Theory from the Synthetic Prescription
Domain: Balancing Bank Teller's Cage
(simplified and modified for the example)
Synthetic Prescription
S → R •
Teller cage
Sort checks from money orders
S → R
Checks
Add on adding machine stamp with teller stamp
S → R
Money orders
Add with pencil, sign with pen
• S → R • S → R • S → R • S → R • S → R • S
Total
Record sum on master sheet
Place cash-out-total record slip in teller machine & press cash out total key
Remove cash out slip and record total on master sheet
Compare sum of checks & money orders with cash out slips total
Sign the master sheet with pen
Cash out balance verifies
Results of Step One
S → R • S → R • S → R • S → R • S → R • S → R • S → R • S
Teller cage
Sort checks and money orders
Cash out evidence prepared
Add totals
Total
Record sum on master sheet
Place cash out record slip in teller machine & press cash out total key
Remove cash out slip and record total on master sheet
Compare sum of checks & money orders with cash out slips total
Sign the master sheet with pen
Cash out balance verifies

Figure 8a

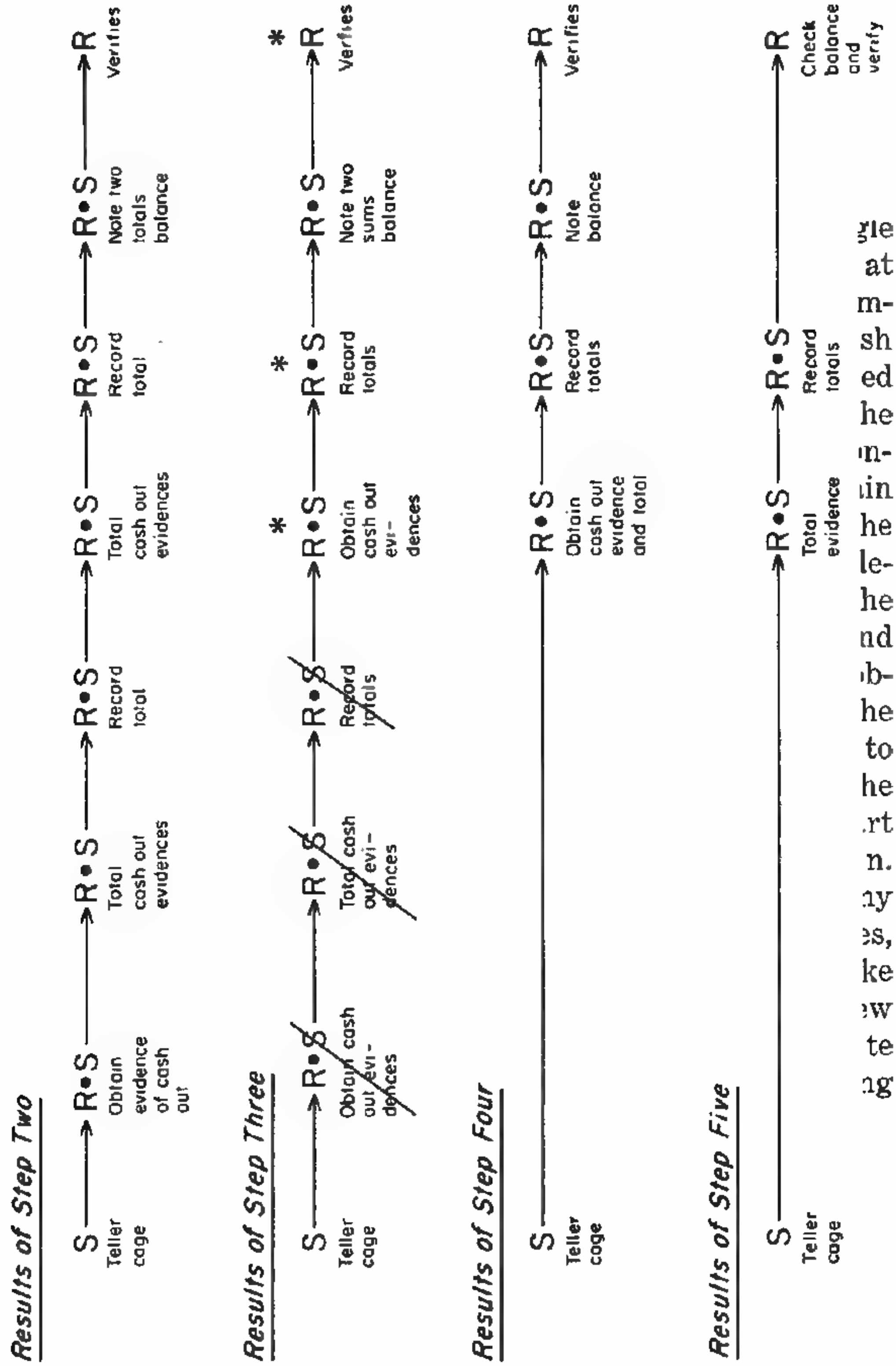

Results of Step Two
S Teller cage
R•S Obtain evidence of cash out
R•S Total cash out evidences
R•S Record total
R•S Total cash out evidences
R•S Record total
R•S Note two totals balance
R Verifies
Results of Step Three
S Teller cage
R•S Obtain cash out evi-dences
R•S Total cash out evi-dences
R•S Record totals
R•S Obtain cash out evi-dences
R•S Record totals
R•S Note two sums balance
R Verifies
Results of Step Four
S Teller cage
R•S Obtain cash out evidence and total
R•S Record totals
R•S Note balance
R Verifies
Results of Step Five
S Teller cage
R•S Total evidence
R•S Record totals
R Check balance and verify

Figure 8b

"non-substitutable" we mean a behavior description that does not specify any particular means of obtaining a result. Adding by adding-machine can be substituted by adding on an abacus).

STEP 3: Combine into a single operant all adjacent operants that bear the same description, and continue until all combinations have been made.

STEP 4: Beginning with the terminal operant in the new prescription, cross out any similar non-adjacent operants, and continue backwards through the chain until you delete all duplications. Star an operant for every duplication.

STEP 5: Rewrite the prescription again, combining wherever possible adjacent operants by the rule of the operant span. Assume that no multiples or generalizations need to be strengthened.

STEP 6: Use these descriptions in the final prescription to write a statement. This is the domain theory.

The prescription used in the illustration is a considerably odified and simplified version of the behavior involved in ılancing a bank teller's cage after the bank is closed. During ıe initial stages of describing this behavior, the matheticist ıay see the task as a hopelessly arbitrary sequence of unılated steps having no underlying logic. At the end of the ɔmain analysis the logic of the behavior becomes quite obious. The bank teller performs two kinds of transactions uring the day: he takes cash in and gives cash out. At the ıd of the day the teller must prove the accuracy of his ·ansactions. He does this by:

(a) Getting the sum of two separate evidences of his cash-out transactions and showing them in balance.
(b) Getting the sum of two separate evidences of his cash-in transactions and showing them in balance.
(c) Getting the sum of two separate evidences of the amount of cash he has on hand and showing them in balance.
(d) Recording and attesting to all sums and balances.

In our example, every operation is for the purpose of obtaining a double evidence for all transactions, and recording and verifying these evidences.

Before considering the manner in which this theory is established in the student's repertory, we shall examine how such a theory might aid the bank teller trainee. Imagine a trainee beginning with the actual operations of cage balancing. He follows his instructions diligently, seeming to understand each operation as the instructor demonstrates it for him; indeed, he is able to repeat the operation after each bit of instruction. But why shouldn't he? There is hardly a single operant in the entire prescription that is not established at appreciable strength in his repertory: he can add, copy numbers, place slips in slots, depress machine keys, distinguish checks from deposit slips, etc., if only, because he has traded at a bank and gone to high school. Nevertheless, when the course of demonstration is run he is quite unable to reconstruct the sequence of balancing the cage. We may complain that his instructor fell down on the job; but imagine that the student is supplied a set of mathetical exercises that are designed to move him through the chain. Properly designed, the exercises will assure his success on every trial. At the end he can perform all the operations correctly; however, he probably has very little sense of what he is doing or why. The great similarity of various operations is a constant threat to his retention, he will have to mentally back up through the chain time after time to get the familiar mental running start that is so characteristic of the performance of a long chain. Imagine that during this new state of mastery the company makes a radical change in the forms and procedures it uses, changes that simplify the operations; but no form looks like the old ones, some operations are reversed, and some new forms for one operation look like the old forms for opposite operations. Our student must not only begin his training again, he is at a disadvantage since he learned habits that now actually compete with efficient performance.

Now it should become clear how the "double evidence" domain theory might have served this student. Given a firm grasp of the theory, he puts it to use immediately. In the early stages of mastery he completes a total and records it, but for a moment he forgets the next step. The theory comes immediately to his aid, he does not have to back up through the chain. He simply says to himself, "What do I do next? Oh, yes, find the second evidence for cash-in." This verbalization is adequate stimulus for him to look selectively among all

the varied events that surround him and locate the cash-in-total slip. A simple verbal chain repeats itself, "CO total one, CO total two, record, check, verify—CI total one, CI total two . . . . . etc." until the operations are complete. The company changes forms and procedures, but the basic logic is the same. Not only can he avoid the total retraining program, but he may be able to master the revised procedure simply by bringing forth his analytic repertory: "find the second evidence for cash in; this form looks like the old cash-in form, but it is a cash-out slip; look for the slip with cash-in printed on it. . . ." It is in this way that theory, as a repertory of selective observing behavior, can produce that great flexibility, speed, accuracy, and resistance to forgetting that is characteristic of old pros in every line of endeavor. It was such a simple theory that guided Ty Cobb to select, from a mass of motion, certain eye and body movements that made his baseball skill seem uncanny to the opposition. The occasional youth who seems blessed with a genius that allows him to bypass the long years of slow accretion of experience, has somewhere learned where to look at the right time and to ignore the myriad irrelevant details.

*Constructing the Analytic Repertory*

It is not sufficient to state the domain theory to a student. Even if he memorized all the words of the theory and displayed confidence in understanding what it was all about, he may be no closer to mastery than before. An example will illustrate the pitfalls to which the mastery of verbal behavior is liable.

A copy editor uses a set of conventional symbols to indicate errors and corrections in copy going to a printer. For all the confusing variety of marks, a domain analysis makes the theory of copy marking quite simple. The copy editor performs sequentially three non-substitutable classes of behavior: he indicates with an appropriate symbol an error in the text, he appropriately symbolizes the correction in the margin, and he separates one correction symbol from another. All other acts are substitutable particulars. Prior to teaching a student to synthesize good copy marking, we might try to get him to understand what the mastery behavior is all about by establishing the verbal repertory that is the domain theory of copy

marking. Our student listens (or reads) and,nods with every evidence of understanding. He is able to repeat the theory when we question him. We then ask him to perform a synthetic step of mastery: find an error of omission in a text and indicate it with a caret. He reads the text and somewhere in the middle he inserts a caret, but we cannot see an error there. When questioned, the student reveals that he did not understand that we meant grammatical errors. His caret marked the author's failure to mention a certain military officer who was also involved in the event the author is describing. The student has missed the entire point of the subject-matter we are trying to teach him, even though he appeared to have such complete understanding. The lesson is clear; words are the most powerful and tricky stimuli man is subject to. For all a student's ability to emit the verbal behavior correlated with mastery (whether the mastery behavior be verbal or nonverbal), we must test the effects of these words before we can be assured that understanding exists.

Since the domain theory is not an intrinsic part of the *synthesis* of mastery, the procedure for teaching it is somewhat different. Analytic behavior is, for the most part, covert; it is a repertory of observing-behavior. The exercises designed to establish the analytic skill have two purposes. (a) First, they should strengthen the appropriate selective observation; (b) second, they should insert the mediating verbal stimuli of the theory into the chain of mastery. Figure 9 gives a description of a verbal "theory chain" in its relation to an analytic chain for a simple subject, copy-marking.

The first initial exercises sequence in the analytic lesson exists for the purpose of increasing the strength of the selective observing behavior, and for bringing this selective observation under increased control of the verbal stimuli in the domain theory. This sequence is called the *induction,* and it usually consists of one to three exercises. The induction supplies materials, simulated or real, that constitute the tools and the context in which mastery is practiced. The first exercise describes the essential behavior of mastery, using the language of the domain theory, and requires the student to follow the example(s) as given: the description of the domain is made in the order in which the chain of mastery occurs. At each step in the description, the exercise requires the student to locate and identify ($S^O$) the essential properties of the stimulus

($S^D$), and then requires him to follow a description of the mastery response, either illustrating the response or its consequences ($S^I$). All materials, stimuli and response illustrations are presented in a schematic form that removes as much confusion and unnecessary detail as possible. The important thing is to get the student to make the relevant observing responses. Overt responses needn't be required of the student since they are not inherent in mastery; however, it is almost certain that the typical student will not follow the verbal description and make the appropriate observations unless he is forced to do so by some technique. (Techniques found useful for this and other purposes will be detailed in a treatise to follow.)

The second exercise of the induction, if it is needed, will repeat the performance of the first; however, the language used for the description will be reduced, as much as is possible, to that of the domain theory. The third exercise restricts its language entirely to that of the domain theory.

The number of exercises required in the induction depends upon the strength in the initial repertory of the observing operants and of the control of the observing operants by the language of the domain theory. Since a domain theory is a repertory of verbal behavior serving to mediate selective observation, its usefulness lies in the mediational function. Figure 9 represents a simple case of the verbal sequence (theory) mediating an analytic behavior sequence. As a system inserted to mediate analytic behavior, the verbal behaviors representing the domain theory normally can be expected to fade as the mastery repertory becomes stronger; in the earlier example of the color-code, the word *penny* loses strength as *brown*

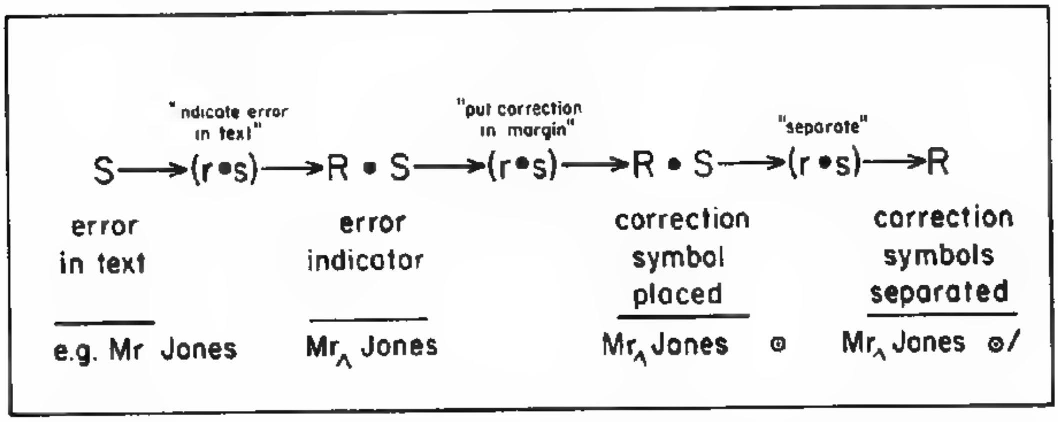



Figure 9. An analytic chain from lesson on proofreading. Showing how a verbal chain from the domain theory mediates the analytic behavior.

becomes a strong determinant of *one*. We are required to bring this verbal repertory only to sufficient strength to call upon during the early stages of constructing the behavior of mastery. The language of the domain theory will be the most important mediational system we can use during training.

The analytic exercises follow the induction. They are designed by the same procedure as the synthetic exercises. The analytic prescription serves as an adequate lesson plan in most cases, and the domain theory language represents the desirable guiding stimuli ($S^{O}$ and $S^{I}$). The analytic exercises require actual mastery performance with as much detail removed as possible. The student is guided through the simplified chain of mastery as described in the analytic prescription, and he performs, with direction, a model problem. These exercises should accomplish three results: (a) establishment of the domain language as mediating behavior; (b) strengthening the sequence of selective looking behavior; and (c) establishment of response conventions and methods of using the lesson materials themselves. Again, it is unnecessary to bring the analytic repertory to the full strength that we desire of final mastery, because the student will have repeated occasion to practice this behavior in the synthetic exercises that follow.

The first step in constructing a lesson plan for the analytic exercises is selecting a representative performance—a *model problem*. The model problem should, as nearly as possible, have these characteristics.

(a) It should be a problem that includes all the operants in the analytic prescription. This is called an *archetype* problem. If there exists no convenient archetype problem, select as few problems as necessary to represent all operants in the analytic prescription. These are called *paradigm* problems.

(b) The model problem should represent performance which is, as much as possible, in the student's initial repertory. Said another way, it should be a problem as familiar as possible.

(c) It should be as simple as possible. Wherever it can be done, the problem should be further simplified so that repetitive operants occur no more than necessary.

It is quite possible, and sometimes desirable, to design an artificial model problem that will best meet the above criteria. In copy proofreading, we could have used an $x$ mark for both the text and the margin symbols. The criteria of familiarity and simplicity can sometimes be served by the choice of a *homologous* problem; less often and with greater risk of misleading by an *analogous* problem. Analogies would be homologies if anologies did not differ in some fundamental way from their analogues.

Analogies need to be avoided only if their use threatens to induce any observing behavior incompatible with mastery as prescribed. It seems to me, as a rule of thumb, that the burden of defense should be upon the use of analogies in systematic design of education. Mastery problems and their homologues exist in enough quantity if we but look for them. Analogies are great conveniences to the teacher who must think quickly on his feet; their difficulties are more likely to be seen by the teacher who is around to watch. Over-generalization, however, (and what else is the over-extension of analogy?) creates great resistance to change. A technology of education provides a leisure that does not require the rapid convenience of analogy; it lacks the benefits of the teacher's personal opportunities to check the pitfalls of analogy.

This suspicion of analogy should be limited to the analytic repertory. The danger of analogy lies in its invitation to excessive generalization—a danger the more to be watched in that part of a lesson where the strengthening of generalization is the primary aim. The analytic exercises are designed particularly to get the student to see what the domain is all about "in a nut shell"—to attain a view of the domain as a whole. Where so much synthetic behavior will be subject to the control of the stimuli produced by this panoramic observation, a glance slightly off-center, a single irrelevant event attended to, threatens the conception of the whole. When taking the student through the individual synthetic acts, however, the danger of over-generalization gives way to the almost certain trend to over-particularization; here analogy need not be feared. Indeed, the use of mediational acts is, in principle, nothing less than the use of analogy: *brown* is like *penny*, *penny* like *one*.

Some will see this danger of analogies as the fair cost of new discovery. Yesterday's most strained analogy is often

today's newest revelation. On occasions all too rare comes a student possessed of an intransigent ineptness, who can neither blend the contradictions hidden in analogies or fully diagnose the mixture his mind is fed; this one, in his efforts to come to peace, turns the opposing parts around and about until by some magic, so obvious when we see it, he transforms the incompatible into unity. Such magic as this we call creativity, born of the subtle contradictions of the analogies men have learned to live with.

## PART V. CHARACTERIZING THE PRESCRIPTION

The prescription, in addition to detailing the repertory that is to be established, depicts many of its gross behavior characteristics: the complex of chains and multiples can be seen at a glance. These behavior characteristics, and not the content of the subject-matter, determine the optimal strategies of teaching. But recognizing chains and multiples is not sufficient; the next stage in mathetics is a more detailed characterizing of the prescription. From this *characterization* we shall be able to plot our strategies and devise a plan for the lessons. The characterization is the most technical and difficult process in mathetics, and I shall describe here only the more important steps in this outline of the system.

Four kinds of information are sought in the characterization of a mastery prescription.

(a) A specification of the generalizations (concepts) that must be established in teaching: this is called a *generalization analysis*.

(b) A specification of existing behavior that threatens to compete with the learning and retention of the new repertory: this is *competition analysis*.

(c) An assessment of the differential beginning strength of the several operants in the domain, and the manner in which they will tend to compete with and facilitate each other during the course of learning. This is called the *interaction analysis*.

(d) An analysis of the initial repertory for the availability of any existing behavior that can be used to help establish and maintain the mastery repertory; this is the *mediation analysis*.

*Generalization Analysis*

Suppose we design lessons to teach clerk trainees how to alphabetize according to filing conventions. In the prescription of mastery filing this operant is designated:

| S ———————→ | R |
|---|---|
| All spaces between filing reference names | count as one alphabetic position weighted less than *A* |

In other words, TROLLO RANGES, Inc. is filed after TROLL ORGANGES, Inc.; the sixth position in the first name is a letter, in the second name it is a space. This would appear to be a simple operant to establish if it is not already at strength; however, the stimulus instance used is only one of many instances. Examine these pairs:

| | | |
|---|---|---|
| SOUTH EASTERN | JOHNS COMPANY | ABBOTT CO. |
| SOUTH WARDLAW | JOHN'S POTATOES | A Z Y PRODUCTS |

In every pair the second name should come before the first: the space in SOUTH EASTERN is not counted as a space *between* reference names; the *'S* in John's is counted as part of the place between reference names and not part of the name. The initials A Z Y are counted as three separate reference names. It should be clear that if we strengthen a single instance of the operant, we cannot safely assume that the correct response will be made when a second stimulus instance occurs. A generalization must be established.

Determining whether there is a generalization to establish and identifying its elements is the first stage in characterization:

(a) Take two separate instances of the stimulus as different from each other as you can find. Estimate: if one instance becomes a strong occasion for the response, will the other instance be adequate to occasion the response when it appears the first time? If no, then you have at least two elements of a generalization: if yes, you have none.

(b) Take a third instance as different from the first two as you can think of. Estimate: if the response is brought under full control of the first two instances, will the third instance have the power to evoke it? If no, you have located a third element of the generalization. You continue in this manner until you cannot discover a new element.

(c) In the remainder of the characterization you treat each generalization element as you would a separate stimulus. The completion of the generalization analysis occurs at another stage.

*Competition Analysis*

It is fundamental that an operant can be brought to strength in one exercise and that it will remain at strength provided (a) extinction does not occur, and (b) competitive behavior does not interfere with it. Of the two, competition presents by far the larger problem in teaching.

The psychologist states that the optimal conditions for behavior competition are found when the animal must make new responses to old, familiar stimuli. This account of competition (or negative transfer) is, however, a misleading one. The topography of the responses of using a door knocker and of saying hello are quite different, but both can be brought under control of the same stimulus without interfering with each other. Negative transfer may take place when new responses are required in familiar situations, but only if both new and old responses involve a competing topography; the responses must be incompatible with one another. *The optimal conditions for competition (or negative transfer) occur when we attempt to bring two topographically incompatible responses under the control of similar stimulus conditions and where one of these responses does not generate a stimulus of high strength for the other response.* If one learned to turn a key to the left and then must learn to turn it to the right, competition will be great. Competition, not disuse, is the cause of forgetting and negative transfer. If competition did not prevail, the mathetical exercise model would be the only teaching strategy necessary.

There are four general sources of competition, and they are illustrated in Figure 10 by a stimulus-response model. In

the illustration, capital letters represent behavior within the mastery repertory we are trying to teach and lower case letters represent behaviors that are not part of the mastery repertory. Strength of operants will be indicated by the lines between S and R; a double line indicates full strength, a single line means that the stimulus is not adequate to evoke the response. The subscript $x$ indicates incompatibility.

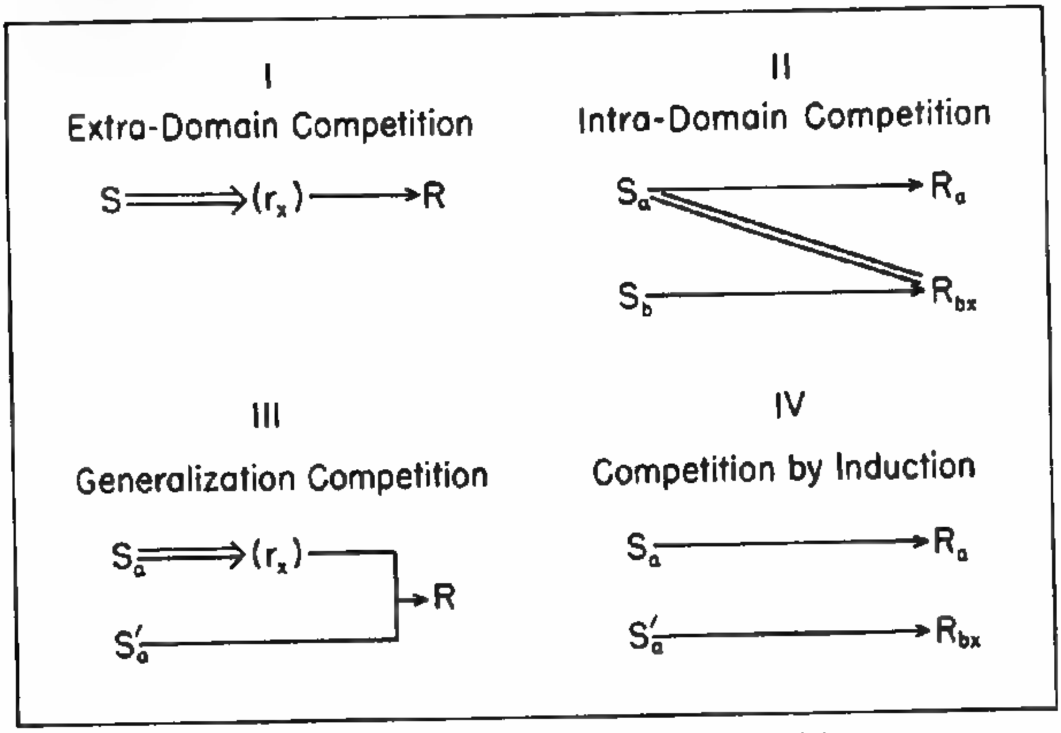



Figure 10. Models of four kinds of competition.

The models in Figure 10 may be read like this:

I. The stimulus is a strong occasion for some response incompatible with mastery.

II. The student must learn to make $R_a$ when he sees $S_a$ and to make $R_{bx}$ when he sees $S_b$; but he has already learned to make $R_{bx}$ when he sees $S_a$, and $R_{bx}$ is incompatible with $R_a$.

III. Both $S'_a$ and $S_a$ should come to evoke R, but $S'_a$ now evokes a response incompatible with R: the generalization will not occur by reinforcing R in the presence of $S_a$.

IV. The student must learn to make $R_a$ to $S_a$, and $R_{bx}$ to $S'_a$, but the two Ss are very similar and the two Rs are incompatible; either operant will compete with the other by induction.

The procedure for assessing the competition with mastery is the second stage of characterization:

(a) Ignore the prescribed response and examine each stimulus and generalization element in turn. Estimate: is there now existing in the repertory of the target student some response under control of this stimulus. If yes, ask if the response is incompatible with the prescribed response. Note all instances and note whether the competing response is relevant to some other prescribed stimulus; if it is, the competition is *intra-domain.*

(b) By each operant in the prescription, note all other responses in the domain that compete with it and all those that it competes with.

This procedure will locate the first three types of competition described in Figure 10. Competition by induction will be revealed in the next stage. It is important to distinguish between these types of competition since each type indicates a different teaching procedure.

*Interaction Analysis*

Competition is not without remedy. There are two general ways in which we can facilitate the strength of a new operant: (a) by *induction* and (b) by *mediation.* Facilitation by induction occurs when *both* the stimulus and response of one operant possess elements in common with *both* the stimulus and response of the new operant; if the first operant is brought to strength, the second will gain strength by induction provided the responses are not incompatible. Similarly, competition can be produced by induction if two stimuli have common elements and their associated responses have incompatible elements. In Figure 11, the lower case letters indicate elements of stimulus and response and the letter $x$ indicates an incompatible topography; the two models represent operants prone to facilitation and competition by induction.

The competitive and facilitative potential of the interactions of operants in a prescribed domain is analyzed in the third stage of characterization:

(a) A matrix is made with the operants of the prescription representing the rows and columns. The first operant in the column is examined first. Compare this operant with each row operant; ask if it will compete with the row operant when brought to strength (use the criteria for competition stated above); if the answer is yes, place a negative sign in the cell.

(b) Ask if the column operant will facilitate the row operant as the former is brought to strength (both competition and facilitation can be induced between the same operants). If yes, place a plus mark in the cell.

(c) Continue until the matrix is exhauted. Significant induction need not be commutative.

With the completion of interaction analysis, the sources of competition have been exhausted. Nothing now remains but to discover elements in the student's existing repertory that will help us overcome the competition and facilitate the establishment of the generalizations and discriminations required for mastery.

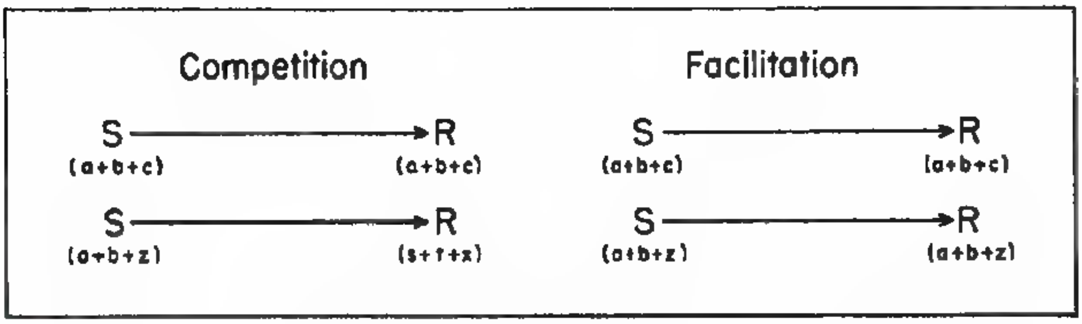



Figure 11. Competition and facilitation by induction.

*Mediation Analysis*

The unusual complexity of the human animal's repertory seems to set him apart from other animals only in degree, but to a degree that has the impact of a difference in principle. The complexity of human behavior can be uniquely accounted for by his verbal repertory; there is no other universal way in which the human animal's ability or capacity excels that of other animals. The great importance of verbal behavior

does not lie in its most immediate effects; these by themselves are essentially useless to man—random motions of the air, and black scribbles on paper. However, the vocal topography is peculiarly suited to the production of a wide variety of sound stimuli and these themselves involve responses that can be made rapidly with minimal energy. Verbal operants can be chained together to guide the human animal to reinforcing manipulations of his environment, even when the environmental stimuli occasioning that reinforcement occurs at a distant time or place. Insofar as verbal operants can perform this mediational service they are of great economic importance to man. Once conditioned, these verbal operants can be stored in the human animal's repertory and preserved for others in libraries. This mediational service spans centuries and continents separating the informing environmental changes from the reinforcing manipulations. It is within this verbal repertory that we find the greatest single source of operants useful for the facilitation of mastery.

But in any system that is designed to use minimal energy to serve so many functions, errors are frequently invited and small errors can be of great consequence. To the extent that the system has the potential for behavior facilitation it has an equal potential for behavior competition.

Whenever we want to establish a new operant (one that has low probability of occurrence in the existing repertory), and wherever there exists little competition with this operant in the existing repertory, the basic mathetical exercise model is a sufficient guide. If the operant we wish to strengthen faces any degree of competition from stronger operants, the exercise model, although essential, will not be sufficient. There exist two general strategies for overcoming this competition. The first is simply to give sufficient reinforcement to the new operant and sufficient extinction to the old to produce the desired strength differential. This well known procedure has persuaded some writers to argue that a good teaching sequence should evoke error responses in the student for purposes of extinction. However, a second strategy exists that, if properly executed, allows us to avoid evoking incorrect responses to bypass the laborious and time-consuming task of extinction.

The second strategy requires the design of appropriate mediational behavior, and this, for the most part, consists of verbal operants. Through mediation we can not only overcome

the competition of incompatible responses, but we can actually make the competition a servant of learning. To understand how this is done, we must have a grasp of three simple behavior principles.

(a) Two similar operants involving incompatible responses will be competitive *if one of these responses does not produce stimulus conditions adequate for the other.* Figure 12 represents, in model, the basic mediating process.

(b) A verbal mediational process is a behavior chain, a chain of verbal operants, whether covert or overt.

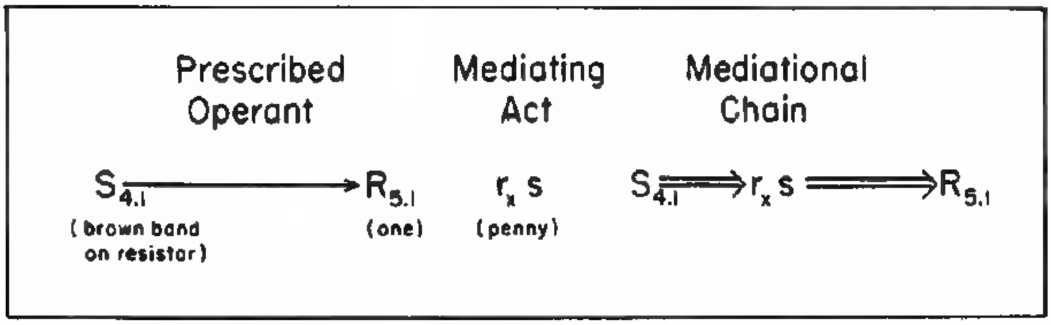



Figure 12. The model of mediation.

(c) There is a tendency for any operant in a chain to weaken and drop out of the repertory when it is not essential for accomplishing the purpose of the chain. Anyone who has chained an animal in the laboratory has observed this short-cutting behavior. The rat who is being trained to pull a lightcord, which produces the $S^D$ for bar pressing, tends to bypass the bar, making only a motion in its direction or barely touching it as he moves to the food pan. We can recognize this property of chaining as one of great economic value for the animal: through these tries at short-cutting he is able to discover unnecessary steps to the goal.

The mediational strategy for reducing behavioral competition can be illustrated in the color code for electrical resistors. The dominant characteristic of this mastery repertory is the multiple discrimination involved in supplying an appropriate number to one of ten colored bands on the resistor. Two forms of competition are potential in establish-

ing these ten operants. Under normal conditions of training the ten operants will compete with each other and, of most importance here, each color stimulus has low strength for evoking the correct number and high strength for evoking many verbal responses incompatible with saying the number. The responses that exist at strength to the simulus *brown* range from *leaves* to *hair*. It is the existence of these many strong operants that makes the color code difficult to learn, and once learned easy to forget. When the matheticist finds in his descriptions of mastery behavior the operants that face such competition he begins to search the target student's repertory for mediational operants. When the major source of competition is estimated to be from operants not relevant to the mastery repertory (extra-domain), we look for some competing response under the control of the prescribed $S^D$, a response that will itself produce a stimulus condition adequate to evoke the response we wish to establish. In the color code we wish to establish the response *one* to the stimulus *brown;* we ask if there is some response to *brown* that is a strong stimulus for *one*. Sometimes we are fortunate enough to discover quickly an appropriate mediator. In this case we find that the operant S (brown) $\longrightarrow$ R (penny) is at high strength in our repertory and that the response *penny* has adequate stimulus strength for the response *one*. Between each $S^D$ and R that we wish to establish we have placed a mediating act; every operant in this chain already exists at high strength. So fortunate we are, in this instance, to discover good mediating acts that we can predict with accuracy that the entire multiple of ten operants can be established in one completed exercise (see Figure 5). The mediating act will drop from the repertory as soon as its services are no longer needed. This is the short-cutting process noted earlier.

This simple example will also serve to illustrate another way in which the analysis can guide educational design. The color code is recognized here as a multiple. Traditionally, however, it has been taught as a chain (using another chain as a mediator); eg, the "rainbow" or the mnemonic sentence, "Black bruins raid our yellow grain; blue violets grow wild." Each word is intended to name or suggest a color, and its order in the sentence gives its number. Repertory analysis prevents us from teaching a behavior aggregate as an unnecessary chain. When this chain is mastered, the student finds it

necessary to say the sentence while counting with his fingers, translate "grow" into "gray" or "raid" into "red", and subtract one from his finger count (the series begins at zero) to get the correct number. When this mediating chain is forgotten, it fails to mediate. Systematic analysis permits us to avoid the pitfall inherent in "clever" mnemonics.

In practice, mediating systems are constructed separately for establishing generalizations and for creating resistance to competition. The purpose of mediation analysis for generalizations is to discover some mediating act that is under the control of many, preferably all, the elements of the stimulus to be generalized. In teaching alphabetizing by filing convention, we want the student to ignore some spaces in file references and to attend to others. In names like VAN DE GRAAF, SOUTH EAST, CO-OPERATIVE, and TWENTY-ONE, we want the student to ignore the spaces; we will attempt to mediate the proper response somewhat like this:

| | | | | |
|---|---|---|---|---|
| (VAN DE GRAAF) | S | | | |
| (TWENTY-ONE) | S | | | |
| (CO-OPERATIVE) | S | → (r · s) | → (r · s) | → R |
| (SOUTH EAST) | S | "this is a single word" | "spaces in single words don't count" | ignore spaces in alphabetizing |

More generally, we are seeking some common factors among the many elements of a stimulus to be generalized—factors for which there exists a common response already at strength.

The analysis of generalization mediation is developed by the following procedure:

(a) Set down all the elements of a stimulus to be generalized. Combine the elements into several groups, each having a common factor. Develop some simple symbols that suggest the property of the common factor.

(b) Set down the several common factor symbols; attempt to group these by finding common factors. Develop a new symbol for the new groups. Continue this process until any further combinations are strained. These groups are called *generalization factors.*

(c) Seek some simple word or statement that is readily suggested by the property common to a group of elements. This will be the verbal mediator.

The procedure for establishing discrimination mediators (to prevent competition) is simpler. For each discriminative stimulus seek some response in the repertory that is already under the control of the stimulus, and that is itself a stimulus with some strength for the prescribed response. This will mediate the generalization factor.

With these procedures the characterization is completed; all that remains is to bring the information together to form a lesson plan.

*Planning the Lesson*

The exercise model is a plan for establishing the strength of a single operant. A lesson plan represents the design of the sequence in which the exercises will occur, describes the mediation that will be used, details the generalization elements and describes specific kinds of competition that must be avoided. The only one of these that we have not considered here is the design of sequential strategies.

Five factors have to be considered in the design of the lesson sequences. They are (a) chains, (b) multiples, (c) generalizations, (d) presence of competition and facilitation, and (e) available mediation. Basic to these five factors are the two purposes of designing a sequence. The first purpose is to make optimal use of the student's motivation; what reinforces him is the observable evidence that he is actually achieving mastery. The second purpose of sequence design is to make optimal use of the interactions between the operants within the domain and between these operants and the initial repertory. By optimal I mean minimizing competition and maximizing facilitation.

*Motivational Determinants of Sequence.* The most important single factor in maintaining the reinforcement value of the exercises is the principle of chaining, which we have considered. By honoring this principle we are starting the student at the terminal acts of mastery, and he is always moving through the chain to that terminus. Allowing him to complete the chain of mastery is the basic way to maintain his motivation.

Sometimes, however, the student will display a distinct weakening of motivation even as he observes himself performing more and more like a master. He is likely to do this when he can say, "I know I'm doing it but I don't understand what I'm doing." This occurs frequently in lessons that do not have an adequate set of analytic exercises. If the analytic exercises are presented prior to the synthetic material, they will have a motivational effect in two ways: They give the student a clearer view of the total amount of work involved, and allow him to assess his gains relative to the whole. Without such a guide, the student may evaluate a significant advance as a minor achievement. The second motivational effect of analytic training is based on the student's justifiable feeling that he will not forget what he is learning now that he understands it.

*Repertory Determinants of Sequence.* We make best use of the available competition and facilitation by following these simple rules of sequence priority.

(a) Everything else being equal, give high preference (in order of presentation) to those operants that produce the greatest facilitation and give low preference to those operants that are most prone to facilitation from others in the domain.

(b) Give high preference to those operants that are most prone to competition from other operants within the domain and low preference to those operants that generate the most competition. (Extra-domain competition has no direct bearing on sequence preference).

(c) If good mediators are available, competition-prone operants need not be given high preference, and those that are facilitation-prone need not be given low preference.

(d) At least one example of each generalization factor should be used. The sequence of the factors should be established by the three rules above.

(e) All operants in a multiple should be established in the same exercise, unless the operants of the multiples are too numerous and the available mediation incomplete.

The reasons for the first two rules should be fairly obvious: you make use of facilitation by first strengthening the operants that generate facilitation, and you give competition-prone operants the strength to resist before exposing them to the effects of competition.

The third rule only affirms the power of mediation to cancel the effects of competition. In theory, complete mediation can be devised for any operant; if a single mediating act is not sufficient, a chain of these acts can be provided. Figures 13a and 13b illustrate the use of a multiple-act mediating chain. The design of specific mediation can, however, present a problem in economic strategy, for the student will supply his own mediation if the lessons do not, and the cost of designing complete mediation will sometimes be great. Left to his devices, the student will usually create mediation of poor quality; but if he has some mediation given him, the quality of his contribution will be raised. Where the cost of design is high and the risks in depending upon the student are not critical, the lessons can provide something less than complete mediation.

The fifth rule, that the entire multiple is best established in one exercise, is dictated by the nature of a multiple discrimination. In a multiple, we have not only to establish the strength of several separate operants, but we must prevent each response from coming under control of the other stimuli in the multiple. Multiples almost always consist of operants with similar stimuli and incompatible responses—the prime conditions for generating competition—and the strengthening of any one operant increases the difficulty of strengthening the others. Where complete mediation is available, it is easy to establish the entire multiple in one exercise (e.g., see the color code exercises in Figure 5). Where the available mediators are not adequate, the multiple can be broken up into sets, each set containing the operants that are most competitive with each other. The *multiple sets* are then taught in successive exercises, but each exercise should require the student to make all the discriminations that were established in earlier exercises. The first exercise demonstrates the first multiple set, the next exercise demonstrates the first two multiple sets, and so on. The order of the sets is determined by the same rules as those for determining the order of individual operants: the first set should contain operants that compete with

## Design of a Mediating Chain for a Large Multiple Discrimination

Domain: Receiving Morse Code

Prescribed Multiple:

$S_{1.1} \longrightarrow R$
("di dah dah dit") (letter p)

$S_{1.2} \longrightarrow R$
("di dah di dit") (letter l)

⋮

$S_{1.33} \longrightarrow R$
("di dah dit") (letter r)

Plan of Mediation for $S_{1.1}$ (letter p)

STEP ONE: *Describe similarities in topographies of the stimulus and response, treating each as an act:*

TOPOGRAPHIES OF BEHAVIOR

S (Code sound from "bug")

(a) Thumb moves "bug"
- —to the right
- —sounds short "dit"
- —"dit" is high pitch (up) ↗

(b) Fingers move "bug"
- —to the left
- —sounds long "dah"
- —"dah" is lower pitch (down) ↙

R (Writing script letter)

(a) Thumb moves pencil
- —to the right
- —short stroke
- —stroke is up ↗

(b) Fingers move pencil
- —to the left
- —long stroke
- —stroke is downward ↙

Figure 13a

STEP TWO: *Find symbolic representation of the similar topographies:*

(a) right-short-up: = "dit"
(b) left-long-down: = "dah"

---

STEP THREE: *Design mediating chain by finding acts that produce the topographic symbols:*

| S ⟶ | (r • s) ⟶ | (r • s) ⟶ | R |
|---|---|---|---|
| "di dah dah dit" | pencil marks left and right | same mark, more like response | letter p |

---

STEP FOUR: *Design exercises based on exercise model and chaining procedure*

Teach entire multiple in each exercise. Require mediating acts to be made overtly; student will later make them covertly as needed

| | | | | |
|---|---|---|---|---|
| *First exercise teaches:* | | | (s) ⟶ | R<br>letter p |
| *Second exercise teaches:* | | (s) ⟶ | (r•s) ⟶ | R<br>p |
| *Third exercise teaches:* | S ⟶<br>"di dah dah dit" | (r • s) ⟶ | | R<br>p |
| *Fourth exercise teaches:* | S ⟶<br>"di dah dah dit" | | | R<br>p |

Figure 13b

each other, but which generate the lowest competition and greatest facilitation with operants in other sets.

The several determinants of order preference are often in conflict. An operant that properly occurs toward the end of a chain may generate great competition with operants occurring earlier in the chain; the order required by the principle of chaining is opposite that required by the principle of discrimination. There is some extent to which the requirements of chaining determine order preference exclusively, but the principle of chaining should not be regarded uncritically. Even where the prescribed repertory consists of a sequence of behavior complexes in which the order of no part can be reversed in mastery performance, we frequently can identify *sub-chains* that can be treated separately, and a late sub-chain can be taught after an earlier one; later the sub-chains can be treated as single operants and chained in the prescribed order.

A sub-chain is, roughly speaking, any distinctive series of acts producing its own unique results. In balancing a teller cage, the bank teller will first prove his cash-out transactions, next the cash-in transactions, and finally compute the total balance. We could teach these three distinctive series (sub-chains) in any order, and then establish a three operant chain taking each (sub-chain) as a single operant. The reader may have had the experience of learning the verses of a long poem in a somewhat random order and finding himself unable to say the poem as a whole; no one verse sets the occasion for the other, although each verse is a well established chain. The poem could now be put in proper order by learning in sequence the first few words of each verse.

There is a more precise definition of a sub-chain; one that provides an efficient basis for the later chaining of the sub-chains. The analytic prescription represents mastery behavior in logically distinctive units; these units are established by stripping away the non-essential, substitutable detail to find the essential nature of the operations constituting mastery. The sub-chains of the synthetic prescription are, then, those chains represented by any complete unit of the analytic prescription. Having a system for recognizing units of the prescription larger than the single operant span, we can now place our several rules for sequencing in proper relation to each other. The following procedure is followed in applying rules to the determination of order:

(a) First, locate all sub-chains in the synthetic prescription. Rule: the retrogressive order required for chaining determines the sequence of exercises for the operants *within* each sub-chain.

(b) Examine each multiple in relation to its available mediation. Estimate how many sets the multiple should be divided into; the estimating bias should be in the direction of establishing multiple sets that are too large, for the same reason that the bias for estimating the operant span should be in the direction of exceeding the span.

(c) Follow the three rules of order that base preference on the generation of competition and facilitation. Establish a preference weight separately for each sub-chain; this is done by totaling the weights for all operants in the sub-chain. These weights determine the preferred order of the sub-chains.

(d) Compute the weights of each generalization factor within a generalization and of each multiple set within a multiple; these weights determine the order preference for the sets within the multiples and the factors within a generalization.

(e) Note: numerical expression of the weights can be made. Each weighted weight should consist of two numbers, a negative and positive value. A positive value is assigned to an operant for every operant that it facilitates and for every operant that is competitive with it. A negative value is given to an operant for every operant it competes with and for every operant that it is facilitated by. No weight is given where available mediation cancels the effects of competition and facilitation.

(f) After all sub-chains are established, chain them in the proper order by establishing a verbal chain using the language of the domain theory that represents each sub-chain.

These steps are summarized in Figures 14a and 14b.

The formal lesson plan is a form that includes all relevant information and procedural guides needed by the exercise writer. It should leave virtually nothing to the imagination of the writer.

## Symbolic Representation of the Steps in Determining the Order of Instruction

### ANALYTIC PRESCRIPTION

$$S_{I} \longrightarrow R \bullet S_{II} \longrightarrow R \bullet S_{III} \longrightarrow R$$

### SYNTHETIC PRESCRIPTION

$$\left.\begin{array}{l} S_{1.1} \to R \\ S_{1.2} \to R \\ S_{1.3} \to R \\ S_{1.4} \to R \end{array}\right\vert \bullet S_2 \longrightarrow R \bullet \left\vert\begin{array}{l} S_{3a} \\ S_{3b} \\ S_{3c} \end{array}\right] \to R \bullet S_4 \longrightarrow R \bullet S_5 \longrightarrow R \bullet S_6 \longrightarrow R \bullet S_7 \longrightarrow R$$

---

STEP ONE: *Obtain weights from interaction analysis*

---

STEP TWO: *Locate sub-chains and order by weight:*

| | | ORDER |
|---|---|---|
| Sub-chain #1 $(S_{I} \longrightarrow R)$: | $\left.\begin{array}{l} S_{1.1} \to R \\ S_{1.2} \to R \\ S_{1.3} \to R \\ S_{1.4} \to R \end{array}\right\vert \bullet S_2 \longrightarrow R \bullet \left.\begin{array}{l} S_{3a} \\ S_{3b} \\ S_{3c} \end{array}\right] \to R$ | 3 |
| Sub-chain #2 $(S_{II} \longrightarrow R)$: | $S_{4.0} \longrightarrow R \bullet S_{5.0} \longrightarrow R$ | 1 |
| Sub-chain #3 $(S_{III} \longrightarrow R)$: | $S_{6.0} \longrightarrow R \bullet S_{7.0} \longrightarrow R$ | 2 |

Figure 14a

## Symbolic Representation (continued)

STEP THREE: *Within each sub-chain, determine multiple sets and weight generalization factors:*

| | | ORDER |
|---|---|---|
| Multiple set # 1 | $S_{I.1} \longrightarrow R$<br>$S_{I.2} \longrightarrow R$ | 1 |
| Multiple set # 2 | $S_{I.1} \longrightarrow R$<br>$S_{I.3} \longrightarrow R$<br>$S_{I.4} \longrightarrow R$ | 2 |
| Generalization factor # 1 | $S_a$ | 3 |
| Generalization factor # 2 | $S_b$ | 1 |
| Generalization factor # 3 | $S_c$ | 2 |

---

STEP FOUR: *Establish order of operants in instruction*

ORDER

1 $S_{III} \longrightarrow R$
2 $S_{II} \longrightarrow R$
3 $S_{I} \longrightarrow R$
4 $S_{5} \longrightarrow R$
5 $S_{4} \longrightarrow R$
6 $S_{7} \longrightarrow R$
7 $S_{6} \longrightarrow R$
8 $S_{b} \longrightarrow R$
9 $S_{c} \longrightarrow R$
10 $S_{a} \longrightarrow R$
11 $S_{2} \longrightarrow R$
12 $S_{I.1} \longrightarrow R$, $S_{I.2} \longrightarrow R$
13 $S_{I.1} \longrightarrow R$, $S_{I.3} \longrightarrow R$, $S_{I.4} \longrightarrow R$
14 ($S_{III} \longrightarrow R$) verbal
15 ($S_{II} \longrightarrow R$) verbal
16 ($S_{I} \longrightarrow R$) verbal

Figure 14b

## GLOSSARY

*Analytic repertory*: The repertory of selective observation developed by the domain theory. It is the behavior usually meant by "understanding" or "theory."

*Archetype problem*: A problem that includes all the operants in the analytic prescription.

*Characterization*: An analysis of the behavior properties of the prescribed repertories, including the generalizations to be taught, the competition that exists with adequate performance, and the behavior available to overcome the competition.

*Differential prescription (D-Px)*: A prescription developed for components of the assumed initial repertory not at full strength in some of the target students. This is part of the mathetical method of handling individual differences.

*Domain theory*: A theory that is uniquely tailored to the assigned subject-matter being analyzed. It is produced to supply a student with a repertory of selective observing behavior, and embraces only the elements of the prescribed synthetic repertory.

*Exercise, Size of*: Every exercise includes as much mastery performance as the student can possibly negotiate, i.e., as much performance as embraces the operant span.

*Generalization element*: A stimulus in a generalization whose response is not occasioned by any other stimulus in the generalization.

*Mathetics*: The systematic application of reinforcement theory to the analysis and reconstruction of those complex behavior repertories usually known as "subject-matter," "knowledge," and "skill."

*Mediator*: An act interposed between the stimulus and response of an operant to insure that the operant will occur until reinforcement gives it sufficient strength: $S \longrightarrow (r \cdot s) \longrightarrow R$.

*Model problem*: A problem developed for the analytic exercises which includes all the operants in the analytic prescription, represents performance from the initial repertory, and is as simple as possible.

*Operant span*: The maximal change in the direction of mastery made possible by one mathetical exercise.

*Paradigm problem*: A problem that requires only some of the operants in the analytic prescription.

*Prescription*: The method of symbolizing a complex of mastery behavior in units of the operant span using the framework of stimulus-response notation.

*Prescription, first approximation (1st Px)*: The first in a series of approximations to the Nth approximation. It describes the step-by-step synthesis of mastery in operant units considerably smaller than the operant span.

*Prescription, second approximation (2nd Px)*: The second in a series of approximations to the Nth approximation. It is developed from the 1st Px and is the first approximation to estimate the operant span.

*Prescription, Nth approximation (Nth Px)*: This is the first confident prescription in a series of prescription approximations. It is not the final prescription.

*Realm theories*: General systems which embrace not only the specific subject-matter, but other matter as well.

*Repertory*: The collecive behaviors existing at strength in an animal at any given time. The *initial repertory* refers to the available behavior prior to instruction, the mastery (or *terminal*) *repertory*, to the behaviors constituting skill in the subject-matter.

*Special theories*: Limited systems which embrace only part of a specific subject-matter.

*Synthetic repertory*: The behavior repertory necessary to synthesize the mastery performance. It is roughly equivalent to what is meant by the word "practice" as distinguished from "theory". *See analytic repertory.*

*Theory*: A repertory of selective looking behavior.

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# EFFECTING THE USE OF EFFICIENT STUDY HABITS

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## *The Problem*

SELF-INSTRUCTIONAL materials do not escape the old problem of poor study habits; indeed, they cannot help but emphasize it. No matter how logical and clear the teaching materials, if the student does not study there is no teaching at all. Even instruction in "effective study habits" is not free of this iron law. Such excellent analyses of effective study habits as Robinson's (1946) are useful only to the extent that they can be transmitted to students. It is one thing for the student to listen and follow the exposition of effective study habits; it is another thing for him to put these habits to use outside the classroom. And no matter how successful self-instructional materials may eventually prove, schools will for some time to come depend upon traditional assignments and texts. They will depend on them long enough to justify attention to improving the manner in which students use available methods of learning.

The work on effective study habits to date represents a considerable array of reasonably sound advice to students. Robinson's Survey Q3R method, as an example, is a system of study far superior to those commonly used by students. Although further analysis could probably improve on the Survey Q3R method, it would be sufficient accomplishment to put this system into wide use. Typical methods for study habit instruction in colleges suffer several disadvantages. They usually require a trained counselor or teacher who must work with small classes to be effective. If every student who could significantly gain from it were enrolled in a remedial study section, a medium-sized college might require about 300 semester hours of trained instructor time each year. The typical college is not in a position to reach all students. Moreover, such courses seldom have the reputation for success necessary to induce

students to enroll. Extraneous motivation is usually required to fill up even existing sections. In addition, the urgent needs of borderline students reduce the time devoted to better students. In short, such tuition is an extra financial burden to the college, and reaches too few students.

It attests to the importance of the problem that poor study habits are a recurrent concern of educators despite the fact that the problem has been around for a long time. A college could not discharge its responsibility to produce graduates if it adhered to the standards of education that its faculty values. With that responsibility outweighing all others, it is the student, in the final analysis, who sets the standard of education. He sets this standard by his study behavior. Students are the most important instructors in the college. No matter how brilliant the lectures or how good the facilities, most students would fail if they did not teach themselves by study outside the classroom, and it should be conceded that most of them would pass if they studied arduously and received only assignments from their professors. It would seem that a concern for the improvement of college instruction would lead us first to examine the methods of studying. That the student is seldom thought of as an instructor, in the truest sense of the word, is really the most startling indictment of our prevalent educational philosophy. When we see the student as what he is—the primary tool of educational methodology—the problem of establishing effective study habits takes priority over all others faced by a college. The issue of efficient education pivots on the study behavior of the student. Our technical concern for the improvement of education has been for matters tangential to this behavior. Study habit is not merely a small but persistent problem of education—it is a fundamental issue. What I describe and propose here is an interim approach that will make do with the materials already available while improving the student's execution of typical assignments.

The importance of study habits extends beyond the formalities of subject-matter education. The products of poor study habits are both cause and consequence of much of what is called "personal maladjustment." The problem is a frequent occasion for visits of students to college clinic and counseling centers. That the consequences of poor study habits are fundamental expressions of repeated failure should give pause to those who would discount the problem as simply a manifesta-

tion of some more basic disorder. Four years of nagging anxiety combined with either failure or compensating behaviors of cheating, meaningless memorizing (cramming), and disguising ignorance, must have their effect. The problem may have far more relevance to what is called mental hygiene than is commonly believed.

*Some Behaviors Characterizing Poor Study Habits*

The promotion of effective study demands both an analysis of the constituent behaviors and an analysis of the existing behaviors that compete with them. Whereas this analysis has often been provided, a description of these behaviors in the present context will further the understanding of the program described here.

The act of studying, regardless of efficiency, is not usually under adequate stimulus control, either by time or by place. The student may study physics at random occasions and at any place he may happen to be on those occasions. Thus, he is subject to all the interfering behaviors conditioned to those occasions. No one occasion becomes uniquely related to study. Even where the student has established regular places and times for study, the immediately preceding occasion is likely to produce behavior competing with that of going to the place of study. He studies physics in the library at ten o'clock if he can resist the reinforcement involved in having coffee with his friends.

A student's actual study behavior consists largely of acts indirectly relevant to, even competitive with, learning. More than anything else, to study is to read; to read is to peruse written matter as one would peruse a novel or newspaper. To study is to underline passages in a text. But why? Underlining is not the behavior one desires to learn. The underlined passages may be later recognized as important material, but usually we wish the student to reproduce this material, not point to it. To study is to copy into a notebook. But copying behavior itself hardly constitutes education, and generally wastes time.

These common conceptions of study behavior are not only unrelated to effective learning behavior, they more than likely interfere with it. Since the material to be learned is often aversive, and since others agree that copying, reading and underlining constitute study, the student may believe that he

has discharged his responsibility by carrying out these behaviors. Moreover, these behaviors are usually accompanied by just enough learning to maintain them. If he fails a test he commonly complains that unrealistic demands were made of him.

Typical reading behavior is not an efficient learning method. Students will read several pages before they become aware of what they are doing. Even then, they frequently do not know what the chapter is all about. They will press on to new material before they understand necessary concepts that went before. And, of course, they underline and copy. Recitation is minimal and review is merely re-reading that which they did not master the first time.

Many study methods have been developed to cope with these difficulties. These methods, if used, can produce decided improvement. The difficulty with the methods lies hidden in the phrase, "if used." The student can be impressed with the plausibility of a method when it is described to him, as the dog-paddler can be convinced of the superiority of the Australian Crawl. These methods rely heavily on recitation, as they should. However, it is one thing to convince a student that recitation is the essential mechanism of learning, it is another to get him to outline the chapter while sitting on a closed book. Recitation is hard work, and its worthy consequences seem distantly related; the reinforcers of this hard work are considerably delayed. And even if the student has mastered an effective technique, there remains the problem of getting him to study in the first place. This, then, is the first problem facing us: how to provide the conditions that will place the *initiation* of study by students under effective stimulus control. All other developments come to nothing if the control of this first step is missing.

This is what is to be established: (a) placing the initiation of study under stimulus control, (b) making the study occasion an effective stimulus for behaviors similar to what counselors call good study habits, and (c) making it possible to accomplish this at a reasonable cost while using few professionals and reaching many students.

A pilot attempt to apply reinforcement principles to these problems has been conducted. The method was sufficiently successful to justify its serving as a point of departure, at least a beginning which can be experimentally modified. In this

pilot attempt, volunteers were sought from among freshmen and sophomore college students. Students were told that the experimenter believed he had a method that would require them to study only in the day time and would lead to an improvement in grades; evenings and weekends would be free. Five students were chosen: two above and three below the college achievement average. They met with the experimenter individually every day for five to ten minutes. At the outset they were required to give a careful analysis of their schedules, including social activities. Considerable information was obtained about their usual habits of study.

On the first day of counseling, a student who had a free period at ten o'clock, immediately after his physics class, was told that he should begin building his regimen by going to the library every day at that time. The student had indicated previously that he had always intended to do this but never got around to it. He was further instructed to leave on the first floor of the library all his books except those relevant to physics. He was to go to a specified room (little used) and proceed to study physics. If he experienced discomfort or began to day-dream, he should leave the library immediately and join his friends at coffee, see his girl, or do whatever he pleased. He was instructed that he must, under no condition, remain to study further. He was told that once he decided to leave, he was to read one page of the physics text carefully or solve the easiest problem assigned to him and then leave immediately, even though his interest had been renewed. These instructions were written down for him, and it was stressed that he should follow them to the letter. He was told that failure to follow them might result in the counselor's refusal to work with him. On the next day the student was questioned about the details of what he had done, and if he had violated any part of the instructions. He was warned that further violation might lead to the termination of the counseling. After the first day no great difficulty was found in students following the instructions. Each day thereafter the student was told that he was to increase the amount of work he performed after he decided to leave the study room. He was to read one page more than the day before (more or less depending upon the nature of the material). The counselor ignored other courses, telling the student to do as he liked about them so long as he did not study them in the specified room. Gradually the student came to

spend all of the hour studying physics. After the first few days there were no further increments in the amount of material required to study, and the student was told that if he finished this amount (say, three physics problems or five pages of reading) he was free to repeat exactly that amount of work if he wanted to. If he wished, he could repeat that amount again and again as long as he had made an independent decision each time. When this regimen had been established for a full week, the experimenter began scheduling an additional course. A different room was designated, and appropriate hours set aside. Always the experimenter began with the course which gave the student the most trouble, and worked toward his easiest course. Eventually, every course was so scheduled, and the student was spending the whole of one hour each day on each course. The experiment was incomplete, but a regular study schedule had been established. The student was still unable to cover all assignments in the allotted time, and he was told to take care of the extra work in any way he pleased as long as he did not use the rooms set aside for daytime study.

Pause here to analyze what was done. Each of the steps described above was suggested by a simple behavior principle. a) First of all, we were trying to make maximal use of available reinforcers. The physics student, for example, had expressed some anxiety over his inability to live up to his resolution to study each day at ten o'clock. This failure was produced by a combination of the aversive nature of the study and competition with social reinforcers. By requiring him to remain only a short time in the study room, and by giving him the option to leave at will *as a formal requirement of the procedure,* we minimized the aversive conditions of his having to forego completely the competing social engagements and of having to read difficult material for an hour. Possibly of greater importance, we may have reduced aversive features when we made the option to abandon study an approved procedure rather than a display of student irresponsibility. Next, we provided the conditions for positive reinforcement when we saw to it that the student complete an assignment before leaving, even though the assignment was small and dictated by the counselor: a counselor represents University authority in the student's thinking. b) We used the principle of successive approximations by requiring the student to master only a small part of the program of studying before he went on. It is easier

to establish consistent visits to the library than it is to establish this while also trying to get him to remain an hour at hard work. c) We used our knowledge of reinforcement schedules when we broke the study assignment into small parts. The act of studying every set of four pages received reinforcement—probably in the form of satisfaction over the completion of an assignment as well as the earning of a right to leave the situation. In technical terms we put the student on a fixed-ratio schedule of reinforcement, requiring a fixed amount of work to be done for reinforcement. It is well established that such a schedule produces high rates of work and increased resistance to extinction (giving up). If the fixed-ratio schedule is "strained," requiring too much work for a given reinforcement, not only will extinction occur, but the task takes on added aversive character and boredom may be heightened. Study assignments often represent strained ratios, the student having a great deal of work to do for rather vague promises of reinforcement. We have broken up the work into small ratios and depended, in part, on escape from the aversive character of the traditional conditions of study. We have increased the ratio gradually, just as we train laboratory subjects to work at high ratios by slow increments. d) Finally, we have produced these effects under a consistent set of stimuli, taking care that these stimuli serve only as occasions for study—study which terminates in reinforcement. This, in turn, adds to the reinforcement value of the study room itself, according to the principle of conditioned reinforcement.

### *Developing Efficient Study Behavior*

Since this paper is concerned with promoting study behavior using available materials, a gross analysis of study behavior is indicated. We chose to begin with the analysis of Robinson (1946), called the Survey Q3R Technique, making modifications where needed. SQ3R is an abbreviation for "survey," "question," "read," "recite" and "review." "Survey" indicates reading over the bold face and italicized headings in a chapter, and serves to give an overall picture of the material to be studied. It also serves as a one minute warm-up. "Question" means going over the material a second time and formulating questions suggested by headings. This sets the student to look for specific answers while reading. "Reading" refers to reading

without underlining or note-taking. "Recitation" indicates outlining or otherwise reciting the material while the book is closed. *It is during recitation that the student is emitting the essential behavior to be learned*; reading is like watching someone else perform. "Review" can take place in the form of checking the outline against the book or re-reading the chapter, for errors of omission and commission.

Previous experience tells us that it is easy to establish surveying behavior; a bit more difficult to establish questioning; and extremely difficult to establish recitation habits. Reading is like watching someone else do it, and this is easy enough; recitation is composing, and this is work inherently more difficult and less practiced than reading.

In our exploratory attempt to establish effective study techniques in five students, we used the method of successive approximation, just as we had in establishing the behavior of spending time in the study room. Only after attendance in the study room was well established did we instruct him to begin with "survey." We explained how and why the survey was to be done and told him to spend not more than a minute surveying a section. In the next session we had him describe how he surveyed the material, and corrected him where he used the method incorrectly. When "survey" came to be routine, we instructed him to begin the questioning phase. At first, we had him ask only one question for each bold face heading, then gradually required him to ask more and more questions. An approximate time limit was set for this phase. Once "survey" and "question" were well established, we began an analysis of his reading habits (reading tests are useful here). He was instructed to stop all underlining and note taking, and we gave him a time limit to meet—so many pages in five minutes—the speed being a fraction below his average speed for the type of material. He was instructed to read only to answer those questions he had previously asked or those that occurred to him as he read. From time to time, the reading speed standard was increased until he had reached a comfortable but reasonable rate. No attempt was made to make him a speed reader. During this period we began developing "recitation." At first we told him to stop at the end of each section, to close his book and sit on it (literally),[1] and to spend three minutes—*no more*—out-

[1] This is important: it discourages both looking at the book and slumping in the chair.

lining what he had read. When this had been well established we increased the recitation time by about a minute a day until he had reached an optimum. This optimum was established by the experimenter on the basis of the type of material and the number of pages. While the recitation time was being increased, we initiated review training. The student was told to open his book immediately after recitation and rapidly peruse the material for errors of commission. A short period of time was set for this review and was increased each day to a limit. Next, we had him seek errors of omission. Finally, when his original reading time had reached the desired value, and when recitation had become fairly easy, the student was instructed to re-read the whole section for review within a reasonable time limit. Throughout this procedure the student was kept on a schedule in which he made an independent decision after each section to stay and study or to leave and play.

While this description covers only the highlights of the actual work with a typical student, it should serve to exemplify the way to apply the principles of behavior to a complex situation where little control is ordinarily possible. It might be said that we were teaching the student to apply the principles. And this is fundamental: the behavior involved in applying behavior principles is also subject to those principles. We tried to make it easy for him to apply these principles to his own study behavior.

The results of our pilot study were promising. Each of the five students remained with us for the college quarter. During the second quarter they reported the continued use of the method, and all demonstrated a significant improvement in grades: the smallest average rise was one letter grade, the highest was four (one student advanced from an F average to a B average). Four of the students now accomplished all their study in the day, and had evenings and weekends free. One required two hours on Saturday morning to complete his work.

### *Modifications and Extensions*

The pilot study left more to be desired. It appears that greater progress could have been made if training in the SQ3R technique had begun earlier. Variations in these details need to be tried. Improvements will be discovered only by more

intensive and precise investigation of the behavior of a single student. It will be necessary to develop means for obtaining more reliable measures of the students' adherence to schedules, actual time spent in the study room, and so on. Always it must be kept in mind, however, that such an investigation as this one defeats its purpose if it becomes too involved. This is inherently a gross method of improving self-instruction.

Some concern can be directed toward special problems of different subject-matters. For example, in directing the schedules of graduate students preparing for language reading examination, we have achieved some success with little effort and time. Many of these special arrangements can be developed into more efficient procedure with little expense or trouble. A description of the highlights of some work in learning to read French will serve to illustrate what we mean by special applications.

We obtained volunteers from among graduate students who were below the required reading proficiency for French, but capable of making their way through a foreign language with greater ease than a beginner. These students simply were not practicing on a regular schedule, although their resolutions were good. The practice was tiresome and interesting events competed with it. We established a special place for this practice, provided copies of a French Journal, a stop watch, graph paper, pencils, and a ruler. We diagnosed the difficulty as a matter of strained ratios of reinforcement combined with lack of good stimulus control and inadequate information about true progress (which we assumed to represent reinforcement in this case). We attempted to do away with these difficulties with the following procedures.

The students were scheduled at a time convenient to them. They were told to spend approximately ten minutes each day in the practice room, and that our methods would lead them to the required proficiency level within a short time. Each was told that he had to follow the instructions to the letter or lose the privilege of our counsel. We numbered the ordinates of the graph paper to designate the average hourly rate of reading in words per hour; the abscissa represented successive practice trials. A line was drawn parallel to the abscissa and intersecting the ordinate at the rate required by the graduate school, say 400 words an hour. They were instructed to start their stop-watches and begin a written translation of a French

passage. The first passage was 25 words in length. The student translated one passage and recorded his time. A chart enabled him to calculate the hourly rate, and he was instructed to plot this rate on the graph paper provided. He had fulfilled the day's requirement when one passage had been translated, but he was told that he could translate as many more as he liked as long as he spent no more than one hour. After several days, the passages were increased to 50 words and later to 100 words.

These conditions provided all the ingredients for success in getting the student to maintain a steady practice schedule. Very quickly each came to spend the entire hour almost every day, although never required to spend more than ten minutes. The small fixed ratios and the immediate knowledge of results were sufficient to maintain this behavior and to place it under the control of a consistent set of stimulus conditions. The First exercise was a requirement; after that, the curve of progress or regression was a built-in challenge which he did not have to take. Rarely did a student leave the room until he had translated a passage with greater speed than on any previous trial. Thus, he left the room each day strongly reinforced. It was unnecessary to instruct him not to leave until he had exceeded his previous performance. No student was content to reach the standard proficiency mark, and all continued beyond this until continued improvement was difficult to realize. On days of little progress a student would leave early, thereby protecting himself from unknown sources of inefficiency.

### *Semi-automation of Counseling*

The time spent with the student by a professional counselor can be reduced in three ways: by substituting for some of the professional's work a set of written instructions; by using other students wherever possible; and by the use of machinery. A feasible means of exploiting these resources is yet to be worked out. It is possible that each student could be given an IBM card on which he recorded his schedule and other pertinent information. This card, when inserted into a machine, could produce a set of instructions tailor-made to his requirements. The machine would punch the card so that the next insertion would yield the second set of instructions. It may be even more efficient to train students as counselors to carry out this same routine, freeing the professional to devote his time to special

cases. A workable instruction book might be developed that would adequately serve most students.

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# THE CONTROL OF EATING

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ALTHOUGH many investigators have described patterns of eating behavior and reported a wide range of factors related to obesity (1, 2, 5, 9, 10, 11), specific techniques for changing an individual's eating behavior are given little or no attention in published reports, and programs of weight control based on behavioral principles are virtually non-existant. This report is an account of the application of some elementary general principles of reinforcement theory (7) to the analysis of the behavior of the human eater. This theoretical framework of reinforcement was used to analyze actual performances in eating, and particularly self-control of eating. Supplementing the account of this system are descriptions of experimentally developed techniques which should illustrate practical applications of the theoretical principles of self-control.[1]

The theoretical analysis begins with the simple observation that the act of putting food in one's mouth is reinforced and strongly maintained by its immediate consequences: the local effects in the gastro-intestinal system. But excessive eating results in increased body-fat and this is aversive to the individual. The problem is therefore to gain control of the factors which determine how often and how much one eats. An individual will manipulate these variables if the control of eating is reinforcing to him—if he escapes from or avoids the *ultimate aversive consequences of eating* (UAC). Unfortunately for the overeater, the long-term or ultimate aversive consequences of obesity are so postponed as to be ineffective compared with the immediate reinforcement of food in the mouth. Alcoholism is a similar example in which hangover symptoms and the full impact of asocial activity are not suffered until considerable time has elapsed. Realization of self-

[1] The experiments are still underway and will be reported separately by the second and third authors.

control, then, demands an arrangement that will bring the influencing conditions into closer association with the reduction of eating behavior.

The analysis and development of self-control in eating involves four steps:

1. *Determining what variables influence eating.* Almost every known behavioral process is relevant to this. Among these are control of eating by stimuli, effect of food deprivation, chaining, avoidance and escape, prepotent and competing behaviors, conditioned alimentary reflexes, and positive reinforcement (2).
2. *Determining how these variables can be manipulated.* Specification of performances within the repertoire by which the individual can manipulate these variables. One example would be the choice of foods which are weak reinforcers, yet rewarding enough to maintain the behavior of eating them at some low level.
3. *Identifying the unwanted effects (UAC) of overeating.* Avoidance of these is the basic motive for developing the required self-control.
4. *Arranging a method of developing required self-control.* Some of the required performances may call for so drastic a change of behavior that it may be necessary to produce the required repertory in stages by reinforcing successive approximations.

Self-control requires for our purposes a more precise definition than is conveyed by the term "will-power". It refers to some specific performances which will lower the disposition to emit the behavior to be controlled. These performances involve the manipulation of conditions influencing this behavior. A convenient datum for our analysis is the *frequency* of the behavior's occurence. The strength, durability or persistence of the behavior is measured by its frequency. Frequency has the measurement advantage of being a continuous variable. Similarly, the disposition to eat can vary from small to large. The various conditions which the individual himself can manipulate to lessen the frequency of the controlled behavior will be presented in detail in the next section, *Avenues of Self-Control.* The technical problem of generating the self-control performance and maintaining it in

strength will be dealt with in the section *Shaping and Maintaining the Self-Control Performance.*

## Avenues of Self-Control

### *The Ultimate Aversive Consequences*

Avoidance of the ultimate aversive consequences (UAC) of uncontrolled eating is essential in developing performances with which a person may regulate his eating behavior. Self-control is needed because of the time lapse between the act of eating and its UAC. To overcome this time lapse, techniques were sought which would derive a conditioned stimulus from the UAC and apply it at the time the disposition to eat was strong. This is based on the principle that almost any event may become aversive when paired with a known aversive event. Such a conditioned stimulus may be the person's own verbal behavior, if specific training procedures are applied. It is not enough for the subject to *know* what the aversive effect of overeating is, for such knowledge by itself leads only to verbal responses weaker than the food-maintained behavior and may not lessen the strong disposition to eat. Therefore an extensive repertoire must be established so that the subject has under his control large amounts of verbal behavior dealing with the consequences of eating. The continued intensive pairing of facts about the UAC with various kinds of eating performance will make the performances themselves conditioned aversive stimuli. Once a given performance such as eating a piece of pie acquires conditioned aversive properties, any approach to it will produce aversive stimuli. These stimuli will reinforce any self-control because the self-control terminates the aversive stimulus and prevents the uncontrolled act. By such a process, cerain foods like pies, cakes, cokes, doughnuts or candy may become conditioned aversive stimuli, at least until other avenues of control become available.

Before the unwelcome consequences of overeating can be used in developing self-control, they must be identified and developed for the individual. It cannot be assumed that an obese person already has a repertoire about the UAC of eating. In the application of the principles to human subjects

being studied, the development of the UAC was one of the major parts of the practical program.. However, developing a repertoire by which the subjects could create an aversive state of affairs for themselves presents serious technical problems. First, to establish this repertoire, the actual aversive events must be identified for the subject in terms that are meaningful for his daily life. Second, the subject must learn an active verbal repertoire with which he can translate caloric intake into ultimate body fat.

We first disclosed, in great detail, the consequences of uncontrolled eating for each individual. After each subject described anecdotes about UAC in group sessions, we helped each one to develop a fluent verbal repertoire about the relevant aversive consequences. We found that simply *recognizing* the various aversive consequences did not give these subjects an active verbal repertoire which could be invoked immediately and whenever needed. To develop an active repertoire about the UAC, we arranged rehearsals, frequent repetitions, and written examinations. In general, the subjects were unaware of their inability to verbalize the relevant aversive consequences, and were surprised by the poor results of the early written and oral examinations. Verbal descriptions of aversive consequences the subjects had actually experienced were far more compelling than reports of future and statistically probable consequences, such as diabetes, heart disease, high blood pressure, or gall bladder disorder. In other words, descriptions of actual or imagined social rejection, sarcastic treatment, extreme personal sensitivity over excess weight, demeaning inferences concerning professional incompetence or carelessness, or critical references to bodily contours or proportions were much more potent. All of our subjects found their constant and unsuccessful preoccupation with dieting aversive, and any ability to control their own habits highly rewarding.

All of the exercises in this area were designed to develop a strong and vivid repertoire that could be introduced promptly in a wide variety of situations intimately associated with eating and despite a strong inclination to eat. The actual aversive effects of being overweight are largely individual matters which differ widely from person to person. We therefore used group discussions as an aid for each person to discover how her body weight affected her life. The discussion

was guided toward explicit consequences and anecdotes rather than general statements such as "I want to lose weight because I will feel better." We found that after only four or more group sessions, subjects shifted from vague statements such as "I'll look better in clothes" to specific ones such as "My husband made a sarcastic remark about an obese woman who crossed the street as we were driving by." Perhaps, the verbalization of the UAC was too aversive before we had demonstrated that self-control was possible.

*Amplifying the Aversive Consequences of Overeating.* To establish the bad effects of eating more than one's daily requirements, it is necessary that the individual know the metabolic relationships between different kinds of food, general level of activity, and gain or loss of weight. Phrases like "Everything I eat turns to fat" illustrate that the required repertoire is frequently absent. Thorough training should be given in the caloric properties of all of the kinds of foods which the individual will encounter. The aversive effects of eating certain undesirable foodstuffs can be amplified by generating verbal repertories which describe the full consequences of eating them. For example, the subject should be made to recognize that a 400-calorie piece of pie is the caloric equivalent of a large baked potato with butter plus a medium-size steak. The pie is equivalent to one-tenth of a pound of weight gained, and so forth. Again, *knowing* these facts is not at issue. The issue is that a strong-enough repertoire be established, and with enough intraverbal connections, that the UAC behavior will occur with a high probability in a wide enough variety of situations.

An important exercise early in the weight-control program is the identification of the individual's actual food intake. The subject's casual summaries of his daily food intake are likely to be grossly inaccurate. His ability to recognize his actual food intake is improved by an interview technique in which the interviewer probes and prompts him: "What did you have for breakfast?" "How many pieces of toast?" "How many pieces of bread?" "What did you do between ten and eleven in the morning?" "Were you at a snack bar or a restaurant at any point during the day?" "Were you offered any candy at any point?" and so forth.

With the pilot subjects, we leaned most heavily on a written protocol which we used as a basis for individual interviews

about their diets. Each subject kept a complete written account of everything she had eaten, along with calculations of fat, carbohydrate, protein, and numbers of calories. A large part of the early sessions was devoted to problems in recording food intake, such as difficulties in estimating mixed foods like gravies, stews, or sauces.

For the first four weeks of the program, when some simpler kinds of self-control were developed, the subjects' caloric intake was set to maintain a constant weight. We overestimated the maintenance levels, and all subjects gained weight during this month. However, the weight increase proved the relationship between caloric intake and weight change in a situation where the caloric intake was carefully defined. In spite of the weight gain, however, some measure of self-control emerged, particularly in changes in the temporal pattern and regularity of eating.

## Deprivation

The effect of food deprivation may be observed in a pigeon experiment in which the frequency of a pigeon's key pecking, maintained by producing food, is measured as a function of changes in the level of food deprivation. Changes in the level of food deprivation produce continuous changes in the bird's performance over an extremely wide range if we can measure the frequency of the bird's pecking. This frequency of pecking is intuitively close to notions like the bird's disposition to eat, probability of action, or motivation. When a wide range of frequency response can be measured sensitively, the level of deprivation affects the bird's performance continuously, from free-feeding body weight to as low as 65 or 70 per cent of normal body weight. Food deprivation of the order of six to twenty four hours constitutes a very small part of the effective range. The magnitude of food deprivation therefore continues to increase the organism's disposition to emit responses, reinforced by food, long after no further changes occur in gastrointestinal reactions (e.g., hunger pangs) and other conditioned effects of food in the mouth. The hunger pangs, which are ordinarily taken as symptoms of hunger (from which the effect of food deprivation is inferred), are more closely related to the conditioned stimuli accompanying

past reinforcements of eating than to the level of food deprivation. The conditioned reflexes involving the gastrointestinal system occur at relatively low levels of deprivation compared with the effective range of food deprivation in respect to the changes in frequency of operant behavior. There may be a similar lack of correspondence between the tendency to verbalize, introspectively, reports of hunger and the actual disposition to eat. For purposes of developing self-control, the actual performances resulting in food in the mouth are more relevant than the introspective reports of "hunger."

Controlling the rate at which the subject loses weight proves to be a major technique of self-control. For any degree of establishment of a self-control repertoire, there is probably some level of food deprivation which will cause the subject to eat in spite of the self-control behavior. Therefore, a major principle of self-control would be to pace the rate of the subject's weight loss so that the effect of the weight loss on the disposition to eat would be less than the given stage of development of self-control. Many avenues of self-control may be learned without causing any weight loss. Placing the eating behavior under the control of specific stimuli or breaking up the chain of responses usually present in the compulsive eater are examples of this. The former will be discussed below. Breaking up a chain causes the eating performance to become a series of discrete acts which are more easily interrupted than a continuous performance in which each chewing response or each swallow occasions placing the next bit of food on the fork.

If the self-control performances which may be developed are to be useful, they must be maintained by conditions which will be present continuously, even after the weight-control therapy procedures are discontinued. Many unsuccessful crash diet programs illustrate the way in which too rapid a loss of weight produces a level of deprivation and a disposition to eat exceeding the existing self-control. The usual diet involves some program which taps the motivation of the dieter temporarily. For example, slight aversive pressure from the husband or family doctor may produce a rapid loss in weight, perhaps on the order of three to five pounds a week. The effect of the rapid weight loss is a large increase in the disposition to eat which then overcomes the subject's temporary motive.

Limiting the diet to one specific food, such as protein,

probably will produce a heightened disposition to eat other food stuffs regardless of the general weight level. These are the traditional specific hungers. An all-protein diet, for example, even if taken without limit of calories, would probably generate an enormous disposition to eat carbohydrates, sugars, and fats. Therefore, a balanced diet should be maintained and a weight loss brought about by a uniform reduction in amount rather than kind of food.

Although the major effects of food deprivation appear when weight losses are of the order of pounds, the time elapsed since eating would have local effects on the disposition to eat. Local satiation effects may best be used as a limited avenue of self-control by arranging the eating schedules so that the subject ingests a meal or a significant amount of food just before a situation in which the disposition to eat might be unusually strong. An example is a social situation in which eating has frequently occurred in the past or when preferred foods are present. The housewife who eats continuously while preparing dinner can control the disposition to nibble the foods being prepared by shifting the preparation of the dinner meal to the period of time immediately following lunch, when her disposition to eat is lower because she has just eaten.

In the application of the self-control principles to actual exercises, we specified a weight loss of one pound per week and insisted that our subjects adhere to this rate of weight loss even though each of them wanted to cut her diet more stringently in order to lose weight at a greater rate. Different rates of weight loss might possibly be arranged at different stages of development of self-control after more is known about the effectiveness of different avenues of self-control and about the relative effects of weight loss depending upon the initial level.

The continued ingestion of food during a meal provided another variation in level of food deprivation which was used to provide a gradual transition to the final self-control performances. Exercises, such as brief interruptions in eating, were first carried out toward the end of the meal when some satiation had occurred. After the subjects began to learn how to use auxiliary techniques to stop eating and their existing eating patterns began to break down, the exercises were moved progressively toward the early part of the meal, when

their levels of deprivation were higher so that the exercises had to be more difficult.

### *Self-control by Manipulating Stimuli*

The characteristic circumstances when an individual eats will subsequently control his disposition to eat. The process is illustrated by the pigeon whose key pecking produces food only when the key is green and not when it is red. The frequency with which the pigeon pecks the key (reinforced by food) will later depend upon which color is present. Thus, changing the color of the key can arbitrarily increase or decrease the frequency of pecking independently of the level of food deprivation. A frequent factor in the lack of self-control in the obese person may be the large variety of circumstances in which eating occurs. In contrast, a much narrower range of stimuli is present during the more infrequent eating periods of the controlled person. Therefore, the disposition to eat possibly could be decreased by narrowing the range of stimuli which are the occasions for the reinforcement by food. By proper choice of the actual stimuli controlling the eating behavior, it should also be possible to increase the individual's control over these stimuli. There are circumstances when even the pathologically compulsive eater will have a considerably lower disposition to eat for periods of time simply because the environment is novel enough so that eating has never occurred then. Consider, for example, walking in an isolated forest area.

The first step in the development of self-control in this category is to narrow the range of existing stimuli which control eating. The overweight individual eats under a large variety of circumstances. Thus, the problem of self-control is made difficult by the large number of daily occasions which bring the tendency to eat to maximal levels because in the past they have been the occasions when eating has occurred. Two kinds of behavior need to be brought under stimulus control. The first is the elicited reflex effects of food, such as salivation, gastric secretion, and other responses of the gastrointestinal tract. The other involves operant behavior, or the behavior involving the striated musculature of the organism—walking, talking, reaching, cooking, and so forth. In the so-called voluntary behaviors, the major datum is the

frequency of the behavior rather than the magnitude of an elicited reflex, as with the smooth-muscle response of the digestive system. Although these two types of behavioral control are inevitably tied together, their properties are different and they must be distinguished both dynamically and statically. In order to break down the control of eating by the stimuli which have been the characteristic occasions on which eating has been reinforced in the past, the stimuli must occur without the subsequent reinforcement by the food. The process is a direct extrapolation from the extinction of a Pavlovian conditioned response. If the dog is to discontinue salivation on the occasion of the bell, the bell must be presented repeatedly in the situation in which the food no longer follows. The amount of saliva the bell elicits then declines continuously until it reaches near-zero. Similarly, the stimuli characteristic of the preparation of a meal will cease to control large amounts of gastric activity if these stimuli can be made to occur without being followed by food in the mouth. Initially, the stimuli will elicit large amounts of gastric activity; but with continued exposure to these stimuli, the amount of activity will decline continuously until low levels are reached.

Delimiting existing stimulus control of eating may take considerable time because (1) the loss of control by a stimulus is a gradual process, requiring repeated exposure to the relevant stimuli; and (2) it may be a long time before the individual encounters all of the situations in which he has eaten in the past. The sudden temptation of the ex-smoker to light a cigarette when he meets an old friend is an example of the latter kind of control.

Self-control developed under procedures involving very special situations and foods (for example, liquid diets, all-protein diets, or hard-boiled eggs and celery) will be difficult to maintain when the diet circumstances return to normal. The very abrupt shift in eating patterns, kinds of food eaten, and characteristic circumstances surrounding eating will weaken the self-control performances as well as strengthen eating behaviors which were previously in the person's repertoire under the control of the more normal environment. Hence, self-control performances must be developed under circumstances and with foods which are to be the individual's final eating pattern.

*Temporal Control of Eating*

The time of day is an important event controlling eating. With the individual who characteristically eats at regular intervals, gastric activity comes to precede these occasions very closely, and is at low levels elsewhere regardless of levels of deprivation. The same can be said for operant behavior associated with eating, although the order of magnitude of some of the parameters may be different. After the conditioned responses associated with eating are brought closely under the control of a strict temporal pattern, feelings of hunger should disappear except just before meal-time. However, many individuals have no such routine patterns of eating, so that the temporal pattern of eating does not limit the amount of gastro-intestinal activity. The obese person frequently eats in the absence of any gastric activity. A technique of self-control in this category would rigidly specify a temporal pattern of eating and find conditions for adhering to it. As with the gastrointestinal reflexes, this general disposition to engage in operant behaviors reinforced by the ingestion of food can be brought under the control of a temporal pattern of eating, with a resulting lower disposition to eat during the intervals between regular meals. In the early stages of learning self-control, the development of a rigid temporal pattern perhaps should be carried out under conditions in which no weight loss is to be expected and the amount of food, ingested at specified meals, is large enough to minimize the disposition to eat on other occasions. The subsequent maintenance of this temporal pattern of eating when the subject begins to lose weight will depend upon the concurrent action of other categories of self-control performances. The control of eating by temporal factors can also be developed for situations other than the normal routine meals, as, for example, at social gatherings and parties. Because the availability of food is predictable here, early stages of self-control can include arranging a specific time when the eating will occur rather than indeterminate consumption of whatever foods happen to be available.

*The Eating Situation*

As with the temporal properties of eating, the actual characteristics of the eating situation may be used to control

the disposition to eat. However, the stimuli here are clearer and probably exert control of an even larger order of magnitude than that of the temporal pattern. This application of the principle of stimulus control is the same as in the temporal contingency: to arrange that eating occur on limited and narrowly circumscribed occasions and never otherwise. To simplify the development of the stimulus control, eating situations should be associated with stimuli which occur infrequently in the individual's normal activities. For example, an eating place in the home should be chosen so that it is maximally removed from the routine activities of the day. Nor should eating occur together with any other kind of activity such as reading. If reading occurs frequently enough while the subject is eating, then reading will increase the disposition to eat because it has been an occasion on which eating has been reinforced.

*Emphasizing the Stimulus Control*

The occasions characterizing eating can be emphasized by deliberately arranging very obvious stimuli. For example, the subject always eats sitting down at a table which has a napkin, a place setting, and a purple table cloth. The latter makes the situation even more distinctive. In the extreme case, a specific item of clothing might be worn whenever the subject eats. Narrowing the range provides another form of stimulus control. By eating only specific foods in specific places, the disposition to eat when other foods are available will be minimized. This factor will also be discussed under chaining; but the aspect emphasized here is the effect of the foods eaten as one of the elements in the occasion associated with eating. If a subject has eliminated ice cream, candy, and cake from his diet, the sweet shop will have little control over his behavior.

In the actual procedures with subjects, stimulus control was the first avenue of self-control developed. The subjects learned to keep daily diet protocols during the first few meetings and to determine the number of calories necessary to maintain their weight. We restricted eating to three meals a day, eliminated concentrated fats and sugars from the diet, and attempted to bring about an increase in the amount of food taken in at meals, particularly at breakfast, to bring about a normal pattern of eating without any expected weight

loss. For individuals having difficulty in restricting their eating to meals, we arranged a specific and routine extra feeding, as, for example, a glass of milk and a few crackers at bedtime. The extra feeding was to be taken routinely, however, so it did not become a reinforcer for increasing the probability of eating on a wide variety of occasions. No weight loss was attempted until the subjects were successful in eating a normal range of food at meals without any eating at other times. We attempted to create an eating pattern which could be carried out without interruption after the weight-control program was terminated. Our major problems were insufficient protein or excess fat in the diet. None of the subjects ate excessive amounts of carbohydrate except perhaps as candy. However, all subjects had trouble eating a full meal. It was paradoxical that women who joined the program because they could not limit their eating had difficulty in ingesting a maintenance diet at mealtimes. One complained of nausea, another of chest pains, and a third of discomfort from overeating. All of the complaints disappeared in a week, however.

### *Chaining*

Eating is a rough designation for a chain of behavioral sequences culminating in swallowing and the subsequent gastrointestinal reflexes. An illustrative sequence might be as follows: Dressing makes possible leaving the house; leaving the house leads to walking to the store; entering the store is followed by the selection of foods, a basket of food is the occasion for paying the clerk and leaving the store; a bag of groceries at home leads to storing the food; stored food is the occasion for cooking or otherwise preparing the food; the prepared food is the occasion for setting the table and sitting down; the sight of food is the occasion for cutting it with a fork or knife; the dissected food leads to placing food in the mouth; food in the mouth is followed by chewing; and chewing is followed by swallowing. The sequence differs from individual to individual and from time to time, but any selected elements illustrate the process.

Because the frequency of occurrence of the final member of the chain depends on the nature of the earlier members of the eating sequence, some degree of self-control can be

arranged by dealing with the dynamic properties of the eating sequence. The length of the chain of responses leading to swallowing will markedly influence the frequency with which the eating sequence is carried out. The longer the sequence of behaviors in the chain and the more behavior sequences in each member of the chain, the weaker will be the disposition to start the chain. This property of chaining suggests a technique of control which could be useful if used in conjunction with the other avenues of control. By arranging that all of the foods available or accessible require a certain amount of preparation or locomotion, the tendency to eat can be reduced simply because the chain of responses leading to swallowing was lengthened. Keeping food out of areas normally entered, shopping on a day-to-day basis (at a time when the disposition to eat is low), buying foods which are not edible without cooking or other preparation, and placing food in less accessible places are some techniques for weakening the disposition to eat by lengthening a chain. As in some of the avenues of control, this technique would be inadequate under extreme levels of deprivation without additional support from other types of control. The chain must not be lengthened too much, or it might become so weakened that prepotent eating behaviors would occur or the chain shortcircuited.

The actual form of the eating chain in the latter members just before swallowing may be rearranged to reduce the rate of eating. The behavior of swallowing is so strongly reinforced that it could occur very soon after food enters the mouth, without very much chewing. Similarly, the behavior of placing food in the mouth (reinforced by the taste of food) has high strength and occurs as soon as the mouth empties. Many eaters carry out this sequence at a very high rate by reaching for additional food just as soon as food is placed in the mouth and by swallowing while the fork is in transit to the mouth. This analysis is confirmed by the high rate with which many obese people eat compared with that of non-obese eaters.

To reduce the rate of eating and to make it possible for the subject to stop eating at any point, we designed simple exercises to break the chain, particularly the near-final members, so that the occasion for placing food on the fork is swallowing rather than chewing. The new sequence was: food on the fork only after other food is swallowed and the mouth

is empty. These exercises depended on ancillary techniques of control already developed by other techniques of self-control. At the start, the interruptions were only a few seconds; then, they were gradually increased to several minutes. The ability to stop eating at *any* point represents the final effect of nearly all of the other avenues of control; nevertheless, it constitutes a separate technique of control demanding special exercises. In later, more difficult exercises, the subject holds food on a fork for various periods of time without eating. Similarly, chewing is prolonged before swallowing for increasing periods. These exercises are carried out initially at the end of a meal, when the deprivation level is low.

The type of food eaten is of major importance in how reinforcing it would be, and hence how long a chain of responses can be maintained by the food reinforcement. The disposition to eat could be somewhat regulated by a selection of foods in the individual's diet that are sufficiently reinforcing (appetizing, caloric, etc.) to be eaten, but minimally reinforcing so as to minimize the resulting disposition to eat. A certain balance must be achieved; if the foods chosen are so unappetizing or unappealing that their reinforcing effect is negligible, the subject will simply switch to other foods. Also relevant here are the dynamic effects of food deprivation. Foods which are maximally reinforcing should be eaten when the individual is less deprived, and minimally reinforcing foods should be eaten under stronger conditions of deprivation. In other words, the effect of the highly reinforcing foodstuffs on the disposition to eat would be minimized by a lower level of deprivation so that the subject can stop eating more easily. In special cases, the food intake could be increased temporarily in order to minimize the highly reinforcing effect of certain foods. For example, if an individual who is highly reinforced by caloric pastries knows she will be in a situation where such pastries are being served, she could lessen the probability of eating them by increasing her food intake during the preceding meal or by a glass of milk before entering the situation.

### *Prepotent Repertoires*

One way to lessen the disposition to eat is to supplant it by establishing other activities incompatible with eating. In

an extreme case, an apparently large disposition to eat is often due to a behavioral repertoire in which eating appears strong because the rest of the repertoire is weak. Some degree of self-control should be possible if some activity could be maintained at a potentially high strength and circumstances arranged so that the subject could engage in this activity whenever the disposition to eat was strong. An example of such an activity might be telephoning a friend just after breakfast instead of indulging in the customary between-meal nibbling. The use of prepotent repertoires as a technique of control implies a certain amount of control over the prepotent repertoire. In order for these substitutive repertoires to be effective, special attention must be given to methods for strengthening them, particularly when they are needed. For example, instead of reading the newspaper as soon as it arrives, it could be put aside until some time when the peak tendency to eat occurs. Similarly, the telephoning of friends could be postponed in order to keep this behavior at high strength. Such activities occur initially because of independent reinforcement. Another kind of prepotent repertoire may be established by starting some strongly reinforced activity whose reinforcement occurs only if the behavior occurs uninterrupted for a period of time. Examples are washing a floor, going to a movie, taking a bus ride, reading a short story, or going for a walk. Such performances will be prepotent over eating because of the temporary aversive consequences resulting from their interruption. In many cases, the prepotent repertoires physically remove the individual from the place where eating can occur.

The effective use of prepotent repertoires depends upon the development of other avenues of control. Probably no one of these "prepotent" performances would be effective by itself if the disposition to eat were strong. For example, the individual going for a walk could simply stop at a restaurant to eat. Nevertheless, there is still a net advantage, because the supplementary types of self-control needed are relatively easy. For example, compare the disposition to stop at a restaurant during a walk with the disposition to eat in the normal situations when eating usually occurs. If the individual usually eats at home, the tendency to stop at a restaurant and eat will be considerably less than the tendency to eat at home. No explicit training was required in the pilot experiment to establish

self-control by the use of prepotent repertoires; but all of the subjects used them during several phases of the experiment.

Prepotent repertoires may be affected by emotional factors. For example, many persons eat when depressed, affronted, thwarted, or frustrated. In the terms of the functional analysis of eating used here, emotional factors may weaken behaviors other than eating so that eating becomes relatively stronger. Putting food in one's mouth remains a highly reinforcing activity even if the remainder of the individual's repertoire is severely depressed. Eating then occurs because it is less disrupted by the emotional variables depressing the rest of the individual's repertoire.

Eating may interact with emotional factors in more subtle, but nonetheless important, ways, as a mechanism by which a person might escape or avoid emitting verbal behavior which is highly aversive, *e.g.*, thinking about impending circumstances which are highly aversive. Because of its very strong and immediate reinforcement, eating will be prepotent over thinking about anxiety-evoking occurrences. Thus, eating comes to acquire two sources of strength: The immediate reinforcement from food in the mouth, and the reinforcement from postponing or avoiding the aversive consequences of emitting the verbal behavior which the eating supplants. Emotional disturbances will also disrupt the performances by which the individual controls himself, as will any general depression or disturbance of the individual's over-all repertoire. Self-control performances will be especially liable to disruption early in their development, before they become strong and maintained.

The manipulation of factors to minimize the effects of emotional disturbances is a separate topic, involving self-control of variables different from those in eating and thus requiring a separate analysis. The main avenue of control in eating lies in increasing the strength and durability of the self-control performances so that they will remain intact during emotional disturbances. For example, a person who has acquired an active and extensive verbal repertoire about all of the personal aversive consequences of being overweight will be able to emit these behaviors even during some general depression of his behavioral repertoire. The behavior about the ultimate aversive consequences of eating will be even more durable during possibly disrupting situations if it has already

been effective in producing self-control, that is, if the behavior about the UAC has been reinforced effectively by suppression of the disposition to eat.

If existing levels of self-control are certain to break down because of an emotional disturbance, the individual should be trained to plan a controlled increase in food intake. The advantage of explicitly increasing the level of food intake would be that the food would be eaten under controlled conditions, so that stimulus control and other factors of self-control already developed would be maintained, and the effects of absence of progress in self-control would be minimized. Overeating under planned conditions would probably weaken the already developed self-control repertoires less than unplanned or uncontrolled eating.

In many situations, the general depression of an individual's repertoire occurs only for a limited time. Here, the necessary self-control performances would be a manipulation of the physical environment so that food is not available then, or would be the creation of a prepotent environment. The depressed individual who wishes to control his eating goes to a movie, takes a ride on a bus, or goes for a long walk. These activities give time for the emotional states to disappear and simultaneously provide an environment in which eating has not been reinforced very frequently in the individual's past experience. Of course, applications of these techniques of control depend upon the prior achievement of a certain amount of self-control, and probably are some of the most difficult areas of self-control to acquire. Such items would not be attempted at an early stage of the self-control program.

## Shaping and Maintaining the Self-Control Performance

Self-control is a very complex repertoire of performance which cannot be developed all at once. If self-control consists of items of behavior with the same dynamic properties as those of the rest of human behavior, the self-control performance, as a complicated repertoire, must be developed in slow steps. These would begin with some performance already in the individual's repertoire and proceed in successive stages to more complicated performances. With each gain in self-control, the individual has a repertoire from which a new

degree of complex behavior may emerge. Simply "telling" the subject the nature of the performances required for the development of self-control is not a sufficient condition for their development. The situation is analogous to that of a complicated motor or intellectual activity. One cannot explain to the novice how to differentiate an equation in calculus without first establishing a repertoire in algebra. Similarly, as most golfers have learned, no amount of verbal instruction will take the place of slow development of behavior reinforced by its effect on the golf ball. The actual disposition to emit the self-control behavior builds up because it was emitted successfully to reduce the long-term aversive effects of the behavior to be controlled. What is required here is to begin with some performance very close to one in the individual's repertoire, and to arrange circumstances so that those performances have at least some effect on the disposition to eat. The early reinforcement of this initial repertoire by a discernible movement in self-control provides the basis for the subject's continued attendance to the self-control program.

In the development of self-control, the concern is not simply the presence or absence of a self-control performance. A group of behaviors must be built constituting a repertoire that will occur with a sufficient degree of certainty to be maximally effective.

Just as the disposition to eat can vary from near-zero to very large values, the behaviors involved in self-control can also be weak or strong. Whether the individual "knows" what the potential techniques of self-control are, or even can emit them, is not so important as the durability of the self-control repertoire. A set of performances is needed which will occur with high-enough probability despite competition from the individual's other repertoires. The maintaining event for the self-control performance is the reduction in the disposition to eat. The effect of the reinforcement is not an all-or-none matter, and the reinforcing effect of gains in self-control repertoires can be variously small, large, or even intermittent. Uncontrolled eating should not be viewed as a failure in control, but simply as the absence of progress. If the positive aspects of the program are emphasized, as well as the development of specific performances to control the disposition to eat, each small increment in the ability of the subject to control himself will reinforce further participation in the self-control

program. A failure of a self-control performance to prevent eating defines an intermittent schedule of reinforcement of the self-control behavior. It may still continue to maintain the performances, just as any other act that is intermittently reinforced.

Some types of self-control require that old performances disappear rather than a specific repertoire, as, for example, the development of stimulus control, be built. The development of this kind of self-control is largely a function of the number of exposures, without eating, in situations when the individual has eaten in the past. Verbal behavior has only limited relevance here, since it can be little more than a report of what is taking place. Recognizing that the preparations for dinner are increasing the disposition to salivate and eat is of little use in controlling these effects. Extinguishing the effects of these stimuli is an orderly process requiring only exposure to the stimuli and passage of time. However, knowledge of the process might be of use in conjunction with the various avenues of self-control, particularly in respect to emphasizing the stimuli involved. Once the subject recognizes that the extinction of the stimulus control is a slow process, even minor decrements in the extent of the control by the stimuli will provide reinforcements for maintaining the self-control, as, for example, when several days are required for extinction. In the absence of knowledge of the order of magnitude of the course of the process, weakly maintained self-control behaviors might extinguish. In the actual self-control program, noting reductions in the strength of eating behavior during its extinction provides interim reinforcement for the self-control performances.

## Discussion

Traditionally, the development of self-control has been in a framework of classical psychoanalytic and dynamic psychotherapeutic approaches to human behavior. These approaches view self-control in terms of its developmental and dynamic origins and the inner-directed, private forces which sustain, direct, or distort its external manifestations. Prior life experiences are considered in detail through interviewing and related techniques, including analyses of current actions and

attitudes (transference and counter-transference). The focus is on the past to assist the individual in discovering those formative experiences and relationships which have functioned to establish current attitudes and current modes of alleviating anxiety and guilt. A major structural goal of this system is the development of effective insight with increased intellectual freedom and more realistic self-appraisal. A major symptomatic goal is the ultimate reduction of anxiety and guilt, with a resultant diminished need to exploit heroic or uneconomic measures in the control of either or both. A fundamental assumption here is that the human being who becomes sufficiently aware of his personal developmental behavioral determinants and who is sufficiently relieved of neurotic anxiety of guilt, will, by virtue of this achievement, progressively lose his dependence on irrational and restrictive defenses. A corollary is that a healthy behavioral repertoire is potentially available at any time the individual gains relief from the guilt and anxiety of his deviant developmental history. These assumptions are not at all unreasonable for many problems encountered by the clinician, and sometimes appear to be convincingly supported by satisfactory therapeutic outcome. However, there are outstanding exceptions, characterized by certain common behavioral elements. Among these are (1) elaborately ritualized performances; (2) long-standing maintenance of such patterns; and (3) large amounts of strongly maintained and sustained activity. These are the symptoms present in many alcoholic individuals, in all obessive-compulsive neurotics, in many patients with neurotic depressive reactions, in drug addicts, in a variety of schizophrenic patients, and in many individuals with eating disturbances (obese as well as anorexic). Successful and sustained therapeutic improvement is exceptional for all, including the obese (8), however prolonged and insightful the therapeutic experience may be. The kind of functional analysis of behavior proposed here may provide a conception of human behavior as an alternative to the classical psychoanalytically oriented systems.

The terms in such an analysis are the actual performances of the patient and their exact effects on his environment. The frequency of occurrence of the performance is studied as a function of its effect on the environment, and every attempt is made to observe and deal with the relevant performances

rather than with inferred processes. The specificity of the analysis does not mean that the patient must have an intact repertoire by which he deals with the world, and attention can be focused on creating whatever repertoire is necessary. Most of the present report is a presentation of certain practical techniques which can be applied to the problem of uncontrolled eating. The preliminary results of this pilot program are not included as a record of even mediocre success, but rather as a description of the medium within which the specific techniques of control were imparted. A much longer follow-up period and a larger number of cases are necessary to develop a successful program as well as test it. Nor can we now designate these aspects of the program which were effective or ineffective. This report is intended to provide a theoretical and practical model for more structured programs of self-control in eating. We have shown eating habits can be changed in a short-term, small-group-therapy program by the use of the basic principles outlined here. Whether or not the weight losses reported during the first 15 sessions are primarily due to the application of the principles outlined can be determined only by appropriately controlled study experiments. We are not concerned whether one or another program can effect weight loss, since many pharmacologic individual- and group-therapy programs lead to temporary loss of weight, as is generally known. The central issue is the development of self-control in eating which will endure and become an available part of the individual's future repertoire. Most conventional programs do not focus on the eating patterns available to the subject after he has lost weight, nor do they present recognizable techniques for developing such future control. Possible exceptions are in individual programs of psychotherapy which are directed toward an exploration and resolution of the unconscious determinants of eating behavior, and in certain of the conditioned-reflex techniques. Yet, even in these programs, this question remains: Do proper eating habits exist after the individual is free of the relevant disability? The program outlined here has the special advantage of focusing directly and specifically on future eating behavior and of presenting even more specific techniques for bringing this behavior under control. Application of the basic principles requires no special instrumental or technical training and is relatively economical. Slow and controlled weight loss under

relatively high-caloric intake levels minimizes medical and psychological problems.

We cannot state whether the program we carried out is suitable for severely obese individuals (particularly those who have medical or psychiatric complications). Nor can we specify how the technical principles and procedures can be applied to subjects of low educational level or of limited intelligence. A major problem here would undoubtedly be the difficulty of daily and accurate caloric-intake records. Some of these fundamental questions are subjects for future study.

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# THE PROGRESS PLOTTER AS A REINFORCEMENT DEVICE

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TO UNDERTAKE the mastery of a subject is to begin a long voyage. The student's principal reward is not that he is doing well at the time, but that what he is doing advances significant achievement. He needs to know he is progressing, not only from step to step, but with reference to accomplishing final mastery. It is not enough for the student to see that he has negotiated single exercises if these successes appear to be infinitesimal progressions. The method of programmed instruction is to break the material into small parts to enable the student to succeed with the least effort. This tends to mislead, and is based upon an assumption that a successful step is itself reinforcing. But if I set out to travel from coast to coast, I find it small encouragement that I can see one foot set in front of the other in the direction the compass points. It is a long walk, and my small steps make me wish for seven-league boots. It is for this reason that mathetics proposes that an exercise represent the largest step a student can take to mastery. The result is that mathetical exercises are more difficult than programmed frames, but are less boring. It is assumed

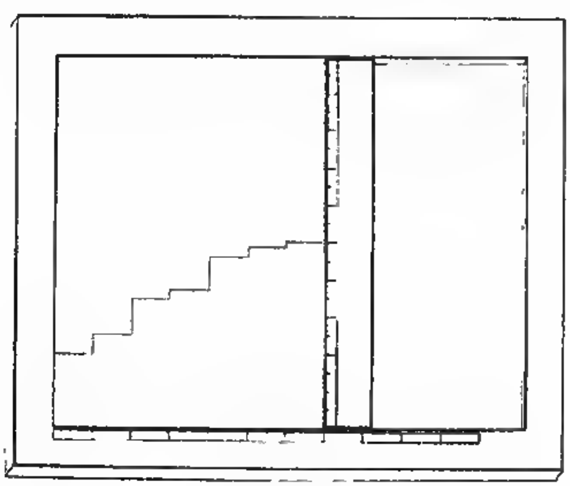

Figure 1. A simple progress plotter.

that the student wants most of all to get to the coast—not to watch his feet mince along the way. Mathetical exercises do not confuse the student any more than programmed frames, but they do make him sweat more. Though a mathetical exercise replaces a mince with seven-league boots, still the untraveled student cannot always know that seven-leagues is much of a step.

Learning accomplishes two things that are often confused with each other: First, it is a change in behavior; second, it is a change in the effect one has on the world. A small behavior change may have a large effect and a large behavior change may have a small effect. A ball player batting .200 begins to bat .400 when he learns to grasp the handle of the bat a little higher. With a small behavior change he has doubled his effectiveness. On the other hand, a student may study law for years and just fail to pass his bar exams. A large behavior change has produced no effect, for he is no more a lawyer now than he was before he began his study.

These two examples illustrate an important distinction students frequently fail to grasp. When the student of Russian has learned the Russian alphabet and its pronunciation, he feels that he has made negligible progress toward final mastery. He is responding to the effect of his accomplishment, for he is no more able to communicate in Russian than he was when he began. He does not realize that he has undergone a large proportion of the total behavior change required to master Russian and have the final effect he is working for. In the language of reinforcement theory, the student is on a "strained ratio"; the ratio of work produced to the effect gained is too large, and the student becomes discouraged (strained). *Fixed ratio schedules of refinforcement,* where the pay-off is scheduled to occur after a certain fixed amount of work lead animals (including man) to work at high and steady rates; however, if the ratio of work to pay-off becomes too high, the animal "strains." In the human, this strain is related to experiences such as boredom and mental fatigue.

Our problem in many areas of learning is to combat this ratio strain and it is not enough to show the student that he is correct after each exercise. What is needed is a system for letting the student see the proportion of total mastery he has accomplished at the end of a set of exercises. Interme-

diate awareness of proportionate accomplishment will serve to put the student on a smaller ratio schedule of reinforcement, thereby reducing the ratio strain.

A simple device, called a progress plotter, has been used to permit the student to see his *rate* of progress toward a mastery criterion. Used in practice or drill sessions, it plots graphically the rising curve of skill. It is effective with study sequences that may be scored for results, such as the number of correct responses per exercise, elapsed time per exercise, number of exercises in a period of time, or any such relative measurement. With the progress plotter the student builds his own curve of achievement as he goes through a lesson sequence. Each rising step of his graph acts as an immediate reinforcement. An occasional plot may disappoint, but the general trend in any series will be upward. With each plot, the amount of gain toward a mastery criterion is easily seen.

An essential requirement of self plotting is simplicity. It has been found that many students are unable to construct graphs on coordinate paper without special training. The progress plotter is, in effect, simply a guiding ruler which plots a score without requiring conscious reference to coordinates. Installations of the device have been found practical among class groups, as well as individuals doing home assignments and studying self-instructional lessons.

In its simplest form, as shown in Figure 1, the progress plotter is a cardboard frame somewhat resembling a schoolchild's old-fashioned slate but holding a fresh sheet of paper on which the progress graph is to be drawn. A vertical ruler is firmly tracked in the frame, top and bottom, so as to slide freely across the surface of the paper while remaining vertical. With the base line of the frame graduated in linear spaces representing successive time periods, and the vertical ruler graduated into the number of exercises from bottom to top, the student may—with the ruler's guidance—place his pencil on the paper at the point of any desired score for any lesson. His actual procedure is to begin with the ruler at the first time period location, which would be at the left end of the frame. At the end of one time period, he marks his first score beside the corresponding score on the vertical ruler. After his second time period, he replaces his pencil on the previous score, slides the ruler one space to the next position, then moves the pencil up to the new score level. Thus his progress

graph is drawn in the form of rising steps as the exercises proceed. At the conclusion of his task—as, indeed, at each stage of it—he sees the graphic pattern of his progress.

Calibrations of the vertical ruler and of the base line are on cardboard strips, removable and replaceable. Appropriate strips may be furnished with each type of lesson. Calibrations may represent, for example, the number of correct items scored for each exercise, the number scored per minute or fraction of a minute, number of seconds or minutes elapsed per exercise, and so forth. Calibrations may be logarithmic or linear as required. The nature of each lesson determines its own coordinate scales.

The student in training is immediately rewarded by actually seeing his accruing progress on a self-drawn graph. On the student's desk, the plotter functions with ease and simplicity and with minimal demand on attention or skill. The plotter has been used with especially good success in teaching rate-drill subjects such as Morse Code.

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